

# TOP QUARK PHYSICS

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## Abstract

In these three lectures, I review the physics of top quark. Each of the lectures is self-contained.

## LECTURE ONE: Global Analysis of the Top Quark Couplings to Gauge Bosons

### 1 Introduction and Motivations

Despite the success of the Standard Model (SM) [1, 2, 3, 4, 5], there is little faith that the SM is the final theory. For instance, the SM contains many arbitrary parameters with no apparent connections [6]. In addition, the SM provides no satisfactory explanation for the symmetry-breaking mechanism which takes place and gives rise to the observed mass spectrum of the gauge bosons and fermions. In this lecture, we study how to use the top quark to probe the origin of the spontaneous symmetry-breaking and the generation of fermion masses.

There are strong experimental and theoretical arguments suggesting the top quark must exist [7]; *e.g.*, from the measurement of the weak isospin quantum number of the left-handed  $b$  quark we know the top quark has to exist. From the direct search at the Tevatron, assuming SM top quark,  $m_t$  has to be larger than 131 GeV [8]. Recently, data were presented by the CDF group at FNAL to support the existence of a heavy top quark with mass  $m_t = 174 \pm 10 (\text{stat}) \pm 12 (\text{syst}) \text{ GeV}$  [9]. Furthermore, studies on radiative corrections concluded that the mass ( $m_t$ ) of a standard top quark has

to be less than 200 GeV [1]. However, there are no compelling reasons to believe that the top quark couplings to light particles should be of the SM nature. Because the top quark is heavy relative to other observed fundamental particles, one expects that any underlying theory at high energy scale  $\Lambda \gg m_t$  will easily reveal itself at low energy through the effective interactions of the top quark to other light particles. Also because the top quark mass is of the order of the Fermi scale  $v = (\sqrt{2}G_F)^{-1/2} = 246$  GeV, which characterizes the electroweak symmetry-breaking scale, the top quark may be a useful tool to probe the symmetry-breaking sector. Since the fermion mass generation can be closely related to the electroweak symmetry-breaking, one expects some residual effects of this breaking to appear in accordance with the generated mass [10, 11]. This means new effects should be more apparent in the top quark sector than any other light sector of the theory. Therefore, it is important to study the top quark system as a direct tool to probe new physics effects [7].

Undoubtedly, any real analysis including the top quark cannot be completed without actually discovering it. In the SM, which is a renormalizable theory, the couplings of the top quark to gauge bosons are fixed by the linear realization of the gauge symmetry  $SU(2)_L \times U(1)_Y$ , although the top quark mass remains a free parameter. However, through the non-linear realization of the gauge symmetry, the couplings of the top quark to gauge bosons can be different from the SM predictions. The goal of this lecture is to study the couplings of the top quark to gauge bosons from the precision data at LEP and examine how to improve our knowledge about the top quark at the current and future colliders. Also we will discuss how to use this knowledge to probe the symmetry-breaking mechanism.

Generally one studies a specific model (*e.g.*, a grand unified theory) valid up to some high energy scale and evolves that theory down to the electroweak scale to compare its predictions with the low energy data [11, 12]. In addition to such a model by model study, one can incorporate new physics effects in a model-independent way formulated in terms of either a set of variables [13, 14, 15, 16] or an effective Lagrangian [17, 18, 19]. In this lecture, we will adopt the latter approach. We simply address the problem in the following way. Assume there is an underlying theory at some high energy scale. How does this theory *appreciably* manifest itself at low energy? Because we do not know the shape of the underlying theory and because a general treatment is usually very complicated, we cannot provide a satisfying answer. Still, one can get some crude answers to this question based on a few *negotiable* arguments suggested by the status of low energy data with the application of the electroweak chiral Lagrangian.

It is generally believed that new physics is likely to come in via processes involving longitudinal gauge bosons (equivalent to Goldstone bosons) and/or heavy fermions such as the top quark. One commonly discussed method to probe the electroweak symmetry sector is to study the interactions among the longitudinal gauge bosons in the TeV region. Tremendous work has been done in the literature [20]. However, this is not the subject of this lecture. As we argued above, the top quark plays an important role in the search for new physics. Because of its heavy mass, new

physics will feel its presence easily and eventually may show up in its couplings to the gauge bosons. If the top quark is a participant in a dynamical symmetry-breaking mechanism, *e.g.*, through the  $\bar{t}t$  condensate (Top Mode Standard Model) [21] which is suggested by the fact that its mass is of the order of the Fermi scale  $v$ , then the top quark is one of the best candidates for search of new physics.

An attempt to study the nonuniversal interactions of the top quark has been carried out in Ref. [10] by Peccei *et al.* However, in that study only the vertex  $t-t-Z$  was considered based on the assumption that this is the only vertex which gains a significant modification due to a speculated dependence of the coupling strength on the fermion mass:  $\kappa_{ij} \leq \mathcal{O}\left(\frac{\sqrt{m_i m_j}}{v}\right)$ , where  $\kappa_{ij}$  parameterizes some new dimensional-four interactions among gauge bosons and fermions  $i$  and  $j$ . However, this is not the only possible pattern of interactions, *e.g.*, in some extended technicolor models [11] one finds that the nonuniversal residual interactions associated with the vertices  $b_L-b_L-Z$ ,  $t_L-t_L-Z$ , and  $t_L-b_L-W$  to be of the same order. In Sec. 4 we discuss the case of the SM with a heavy Higgs boson ( $m_H > m_t$ ) in which we find the size of the nonuniversal effective interactions  $t_L-t_L-Z$  and  $t_L-b_L-W$  to be of the same order but with a negligible  $b_L-b_L-Z$  effect.

Here is the outline of our approach. First, we use the chiral Lagrangian approach [22, 23, 24, 25] to construct the most general  $SU(2)_L \times U(1)_Y$  invariant effective Lagrangian including up to dimension-four operators for the top and bottom quarks. Then we deduce the SM (with and without a scalar Higgs boson) from this Lagrangian, and only consider new physics effects which modify the top quark couplings to gauge bosons and possibly the vertex  $b_L-b_L-Z$ . With this in hand, we perform a comprehensive analysis using precision data from LEP. We include the contributions from the vertex  $t-b-W$  in addition to the vertex  $t-t-Z$ , and discuss the special case of having a comparable size in  $b-b-Z$  as in  $t-t-Z$ . Second, we build an effective model with an approximate custodial symmetry ( $\rho \approx 1$ ) connecting the  $t-t-Z$  and  $t-b-W$  couplings. This reduces the number of parameters in the effective Lagrangian and strengthens its structure and predictability. After examining what we have learned from the LEP data, we study how to improve our knowledge on these couplings at the electron colliders, such as the SLC and the NLC (Next Linear Collider) [26].<sup>1</sup> We will discuss the physics of top quark at hadron colliders, such as the Tevatron and the LHC (Large Hadron Collider), in great details in the third lecture.

The rest of this lecture is organized as follows. In Sec. 2 we provide a brief introduction to the chiral Lagrangian with an emphasis on the top quark sector. In Sec. 3 we present the complete analysis of the top quark interactions with gauge bosons using LEP data for various scenarios of symmetry-breaking mechanism. In Sec. 4 we discuss the heavy Higgs limit ( $m_H > m_t$ ) in the SM model as an example of our proposed effective model at the top quark mass scale. In Sec. 5 we discuss how the electron colliders (such as the SLC and the NLC) can contribute to the measurement of these couplings. Some discussion and conclusions for this lecture are given in Sec. 6.

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<sup>1</sup> We use NLC to represent a generic  $e^-e^+$  supercollider.

## 2 Introduction to the Chiral Lagrangian

The chiral Lagrangian approach has been used in understanding the low energy strong interactions because it can systematically describe the phenomenon of spontaneous symmetry-breaking [22]. Recently, the chiral Lagrangian technique has been widely used in studying the electroweak sector [10, 18, 25, 27, 28, 29, 30, 31], to which this work has been directed.

A chiral Lagrangian can be constructed solely on the symmetry of the theory without assuming any explicit underlying dynamics. Thus, it is the most general effective Lagrangian that can accommodate any truly fundamental theory possessing that symmetry at low energy. Since one is interested in the low energy behavior of such a theory, an expansion in powers of the external momentum is performed in the chiral Lagrangian [23].

In general one starts from a Lie group  $G$  which breaks down spontaneously into a subgroup  $H$ , hence a Goldstone boson for every broken generator is to be introduced [24]. Consider, for example,  $G = SU(2)_L \times U(1)_Y$  and  $H = U(1)_{em}$ . There are three Goldstone bosons generated by this breakdown,  $\phi^a$ ,  $a = 1, 2, 3$  which are eventually eaten by  $W^\pm$  and  $Z$  and become the longitudinal degree of freedom of these gauge bosons.

The Goldstone bosons transform non-linearly under  $G$  but linearly under the subgroup  $H$ . A convenient way to handle this is to introduce the matrix field

$$\Sigma = \exp \left( i \frac{\phi^a \tau^a}{v_a} \right), \quad (1)$$

where  $\tau^a$ ,  $a = 1, 2, 3$ , are the Pauli matrices normalized as  $\text{Tr}(\tau^a \tau^b) = 2\delta_{ab}$ . Because of  $U(1)_{em}$  invariance  $v_1 = v_2 = v$ , but  $v$  is not necessarily equal to  $v_3$ . The matrix field  $\Sigma$  transforms under  $G$  as

$$\Sigma \rightarrow \Sigma' = \exp \left( i \frac{\alpha^a \tau^a}{2} \right) \Sigma \exp \left( -iy \frac{\tau^3}{2} \right), \quad (2)$$

where  $\alpha^{1,2,3}$  and  $y$  are the group parameters of  $G$ .

In the SM, being a special case of the chiral Lagrangian,  $v = 246 \text{ GeV}$  is the vacuum expectation value of the Higgs boson field. Also  $v_3 = v$  arises from the approximate custodial symmetry in the SM. It is this symmetry that is responsible for the tree-level relation

$$\rho = \frac{M_W^2}{M_Z^2 \cos^2 \theta_W} = 1 \quad (3)$$

in the SM, where  $\theta_W$  is the electroweak mixing angle. In this lecture, we assume the full theory guarantees that  $v_1 = v_2 = v_3 = v$ .

Out of the Goldstone bosons and the gauge boson fields one can construct the bosonic gauge invariant terms in the chiral Lagrangian

$$\mathcal{L}_B = -\frac{1}{4} W_{\mu\nu}^a W^{\mu\nu a} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + \frac{1}{4} v^2 \text{Tr}(\mathbf{D}_\mu \Sigma^\dagger \mathbf{D}^\mu \Sigma), \quad (4)$$

where the covariant derivative

$$D_\mu \Sigma = \partial_\mu \Sigma - ig W_\mu^a \frac{\tau^a}{2} \Sigma + ig' \Sigma B_\mu \frac{\tau^3}{2}. \quad (5)$$

In the unitary gauge  $\Sigma = 1$ , one can easily see how the gauge bosons acquire a mass. In Eq. (3),  $M_W = gv/2$  is the mass of  $W_\mu^\pm = (W_\mu^1 \mp iW_\mu^2)/\sqrt{2}$ ,  $M_Z = gv/2/\cos\theta_W$  is the mass of  $Z_\mu = \cos\theta_W W_\mu^3 - \sin\theta_W B_\mu$ . The photon field will be denoted as  $A_\mu = \sin\theta_W W_\mu^3 + \cos\theta_W B_\mu$ .

Fermions can be included in this context by assuming that they transform under  $G = SU(2)_L \times U(1)_Y$  as [27]

$$f \rightarrow f' = e^{iyQ_f} f, \quad (6)$$

where  $Q_f$  is the electromagnetic charge of  $f$ .

Out of the fermion fields  $f_1, f_2$  and the Goldstone bosons matrix field  $\Sigma$  the usual linearly realized fields  $\Psi$  can be constructed. For example, the left-handed fermions  $[SU(2)_L \text{ doublet}]$  are constructed as

$$\Psi_L = \Sigma F_L = \Sigma \begin{pmatrix} f_1 \\ f_2 \end{pmatrix}_L \quad (7)$$

with  $Q_{f_1} - Q_{f_2} = 1$ . One can easily show that  $\Psi_L$  transforms under  $G$  linearly as

$$\Psi_L \rightarrow \Psi'_L = g \Psi_L, \quad (8)$$

where  $g = \exp(i\frac{\alpha^a \tau^a}{2}) \exp(i\frac{y}{2}) \in G$ . Linearly realized right-handed fermions  $\Psi_R$   $[SU(2)_L \text{ singlet}]$  simply coincide with  $F_R$ : *i.e.*,

$$\Psi_R = F_R = \begin{pmatrix} f_1 \\ f_2 \end{pmatrix}_R. \quad (9)$$

Out of those fields with the specified transformations it is straightforward to construct a Lagrangian which is invariant under  $SU(2)_L \times U(1)_Y$ .

Since the interactions among the light fermions and the gauge bosons have been well tested to agree with the SM, we only consider new interactions involving the top and bottom quarks. We ignore all possible mixing of the top quark with light fermions in these new interactions. In case there exists a fourth generation with heavy fermions, there can be a substantial impact on the Cabibbo-Kobayashi-Maskawa (CKM) matrix element  $V_{tb}$ . To be discussed later, this effect is effectively included in the new nonstandard couplings of  $t$ - $b$ - $W$ .

Following Ref. [27], we define

$$\Sigma_\mu^a = -\frac{i}{2} \text{Tr}(\tau^a \Sigma^\dagger D_\mu \Sigma), \quad (10)$$

which transforms under  $G$  as:

$$\Sigma_\mu^3 \rightarrow \Sigma'^3_\mu = \Sigma_\mu^3, \quad (11)$$

$$\Sigma_\mu^\pm \rightarrow \Sigma_\mu^{\prime\pm} = e^{\pm iy} \Sigma_\mu^\pm, \quad (12)$$

where

$$\Sigma_\mu^\pm = \frac{1}{\sqrt{2}}(\Sigma_\mu^1 \mp i\Sigma_\mu^2). \quad (13)$$

In the unitary gauge,  $\Sigma = 1$ , we have

$$\Sigma_\mu^3 = -\frac{1}{2} \frac{gZ_\mu}{\cos \theta_W}, \quad (14)$$

$$\Sigma_\mu^\pm = -\frac{1}{2} g W_\mu^\pm. \quad (15)$$

Consider the interaction terms up to dimension-four for the  $t$  and  $b$  quarks. From Eqs. (7) and (9) we denote

$$F = \begin{pmatrix} t \\ b \end{pmatrix} = F_L + F_R, \quad (16)$$

with  $f_1 = t$  and  $f_2 = b$ . The SM Lagrangian can be deduced from

$$\begin{aligned} \mathcal{L}_0 &= \bar{F} i \gamma^\mu \left( \partial_\mu - i g' \left( \frac{Y}{2} + \frac{\tau^3}{2} \right) B_\mu \right) F - \bar{F} M F \\ &\quad - \bar{F}_L \gamma^\mu \tau^a F_L \Sigma_\mu^a + \mathcal{L}_B, \end{aligned} \quad (17)$$

where  $Y = 1/3$  and  $M$  is a diagonal mass matrix

$$M = \begin{pmatrix} m_t & 0 \\ 0 & m_b \end{pmatrix}. \quad (18)$$

$\mathcal{L}_0$  is invariant under  $G$ , and the electric charge of fermions is given by  $Y/2 + T^3$ , where  $T^3$  is the weak isospin quantum number. Taking advantage of the chiral Lagrangian approach, additional nonstandard interaction terms, invariant under  $G$ , are allowed [27]

$$\begin{aligned} \mathcal{L} &= -\kappa_L^{\text{NC}} \bar{t}_L \gamma^\mu t_L \Sigma_\mu^3 - \kappa_R^{\text{NC}} \bar{t}_R \gamma^\mu t_R \Sigma_\mu^3 \\ &\quad - \sqrt{2} \kappa_L^{\text{CC}} \bar{t}_L \gamma^\mu b_L \Sigma_\mu^+ - \sqrt{2} \kappa_L^{\text{CC}\dagger} \bar{b}_L \gamma^\mu t_L \Sigma_\mu^- \\ &\quad - \sqrt{2} \kappa_R^{\text{CC}} \bar{t}_R \gamma^\mu b_R \Sigma_\mu^+ - \sqrt{2} \kappa_R^{\text{CC}\dagger} \bar{b}_R \gamma^\mu t_R \Sigma_\mu^-, \end{aligned} \quad (19)$$

where  $\kappa_L^{\text{NC}}, \kappa_R^{\text{NC}}$  are two arbitrary real parameters,  $\kappa_L^{\text{CC}}, \kappa_R^{\text{CC}}$  are two arbitrary complex parameters, and the superscripts  $NC$  and  $CC$  denote neutral and charged currents, respectively. In the unitary gauge we derive the following nonstandard terms in the chiral Lagrangian with the symmetry  $\frac{SU(2)_L \times U(1)_Y}{U(1)_{em}}$

$$\begin{aligned} \mathcal{L} &= \frac{g}{4 \cos \theta_W} \bar{t} \left( \kappa_L^{\text{NC}} \gamma^\mu (1 - \gamma_5) + \kappa_R^{\text{NC}} \gamma^\mu (1 + \gamma_5) \right) t Z_\mu \\ &\quad + \frac{g}{2\sqrt{2}} \bar{t} \left( \kappa_L^{\text{CC}} \gamma^\mu (1 - \gamma_5) + \kappa_R^{\text{CC}} \gamma^\mu (1 + \gamma_5) \right) b W_\mu^+ \\ &\quad + \frac{g}{2\sqrt{2}} \bar{b} \left( \kappa_L^{\text{CC}\dagger} \gamma^\mu (1 - \gamma_5) + \kappa_R^{\text{CC}\dagger} \gamma^\mu (1 + \gamma_5) \right) t W_\mu^-. \end{aligned} \quad (20)$$

A few remarks are in order regarding the Lagrangian  $\mathcal{L}$  in Eqs. (19) and (20).

- (1) In principle,  $\mathcal{L}$  can include nonstandard neutral currents  $\overline{b_L}\gamma_\mu b_L$  and  $\overline{b_R}\gamma_\mu b_R$ . For the left-handed neutral current  $\overline{b_L}\gamma_\mu b_L$  we discuss two cases:
  - (a) The effective left-handed vertices  $t_L-t_L-Z$ ,  $t_L-b_L-W$ , and  $b_L-b_L-Z$  are comparable in size as in some extended technicolor models [11]. In this case, the top quark contribution to low energy observables is of higher order through radiative corrections; therefore, its contribution will be suppressed by  $1/16\pi^2$ . In this case, as we will discuss in the next section, the constraints derived from low energy data on the nonstandard couplings are so stringent (of the order of a few percent) that it would be a challenge to directly probe the nonstandard top quark couplings at the Tevatron, the LHC, and the NLC.
  - (b) The effective left-handed vertex  $b_L-b_L-Z$  is small as compared to the  $t-t-Z$  and  $t-b-W$  vertices. We will devote most of this lecture to the case where the vertex  $b_L-b_L-Z$  is not modified by the dynamics of the symmetry-breaking. This assumption leads to interesting conclusions to be seen in the next section. In this case one needs to consider the contributions of the top quark to low energy data through loop effects. A specific model with such properties is given in Sec. 4.
- (2) We shall assume that  $b_R-b_R-Z$  is not modified by the dynamics of the electroweak symmetry-breaking. This is the case in the Extended Technicolor models discussed in Ref. [11]. The model discussed in Sec. 4 is another example.
- (3) The right-handed charged current contribution  $\kappa_R^{\text{CC}}$  in Eqs. (19) and (20) is expected to be suppressed by the bottom quark mass. This can be understood in the following way. If  $b$  is massless ( $m_b = 0$ ), then the left- and right-handed  $b$  fields can be associated with different global  $U(1)$  quantum numbers. ( $U(1)$  is a chiral group, not the hypercharge group.) Since the underlying theory has an exact  $SU(2)_L \times U(1)_Y$  symmetry at high energy, the charged currents are purely left-handed before the symmetry is broken. After the symmetry is spontaneously broken and for a massless  $b$  the  $U(1)$  symmetry associated with  $b_R$  remains exact (chiral invariant) so it is not possible to generate right-handed charged currents. Thus  $\kappa_R^{\text{CC}}$  is usually suppressed by the bottom quark mass although it could be enhanced in some models with a larger group  $G$ , *i.e.*, in models containing additional right-handed gauge bosons.

We find that in the limit of ignoring the bottom quark mass,  $\kappa_R^{\text{CC}}$  does not contribute to low energy data through loop insertion at the order  $m_t^2 \ln \Lambda^2$ , therefore we cannot constrain  $\kappa_R^{\text{CC}}$  from the LEP data. However, at the Tevatron and the LHC  $\kappa_R^{\text{CC}}$  can be measured by studying the direct detection of the top quark and its decays. This will be discussed in the third lecture.

It is worth mentioning that the photon does not participate in the new nonuniversal interactions as described in the chiral Lagrangian  $\mathcal{L}$  in Eq. (20) because the  $U(1)_{em}$  symmetry remains an exact symmetry of the effective theory. Using Ward identities one can show that such nonuniversal terms should not appear. To be precise, any new physics can only contribute to the universal interactions of the photon

to charged fields. This effect can simply be absorbed in redefining the electromagnetic fine structure constant  $\alpha$ , hence no new  $t$ - $t$ - $A$  or  $b$ - $b$ - $A$  interaction terms will appear in the effective Lagrangian after a proper renormalization of  $\alpha$ .

Here is a final note regarding the physical Higgs boson. It is known that the gauge bosons acquire masses through the spontaneous symmetry-breaking mechanism. In the chiral Lagrangian this can be seen from the last term in  $\mathcal{L}_B$  (see Eq. (4)), which only involves the gauge bosons and the unphysical Goldstone bosons. This indicates that the chiral Lagrangian can account for the mass generation of the gauge bosons without the actual details of the symmetry-breaking mechanism. Furthermore, the fermion mass term is also allowed in the chiral Lagrangian,

$$- m_{f_i} \bar{f}_i f_i, \quad (21)$$

because it is invariant under  $G$ , where the fermion field  $f_i$  transforms as in Eq. (6).

From this it is clear the Higgs boson is not necessary in constructing the low energy effective Lagrangian. Indicating that the SM Higgs mechanism is just one example of the possible spontaneous symmetry-breaking scenarios which might take place in nature. Still, a Higgs boson can be inserted in the chiral Lagrangian as an additional field ( $SU(2)_L \times U(1)_Y$  singlet) with arbitrary couplings to the rest of the fields. To retrieve the SM Higgs boson contribution at tree level, one can simply substitute the fermion mass  $m_f$  by  $g_f v$  and  $v$  by  $v + H$ , where  $g_f$  is the Yukawa coupling for fermion  $f$  and  $H$  is the Higgs boson field. Hence, we get the scalar sector Lagrangian

$$\mathcal{L}_H = \frac{1}{2} \partial_\mu H \partial^\mu H - \frac{1}{2} m_H^2 H^2 - V(H) + \frac{1}{2} v H \text{Tr} \left( D_\mu \Sigma^\dagger D^\mu \Sigma \right) + \frac{1}{4} H^2 \text{Tr} \left( D_\mu \Sigma^\dagger D^\mu \Sigma \right), \quad (22)$$

where  $V(H)$  describes the Higgs boson self-interaction. The coefficients of the last two terms in the above equation can be arbitrary for a chiral Lagrangian with a scalar field other than the SM Higgs boson. Similarly, the coupling of  $f_i$  and the scalar (Higgs) boson  $H$  can in general be written as

$$- c_i \frac{m_{f_i}}{v} H \bar{f}_i f_i, \quad (23)$$

where  $c_i = 1$  for the SM Higgs boson. In this analysis we will discuss models with and without a Higgs boson. In the case of a light Higgs boson ( $m_H < m_t$ ) we will include the Higgs boson field in the chiral Lagrangian as a part of the light fields with no new physics being associated with it. In the case of a heavy Higgs boson ( $m_H > m_t$ ) in the full theory, we assume the Higgs boson field has been integrated out and its effect on low energy physics can be thought of as a new heavy physics effect which is already included in the effective couplings of the top quark at the scale of  $m_t$ . Finally, we will consider the possibility of a spontaneous symmetry-breaking scenario without including a SM Higgs boson in the full theory. In this case we consider the effects on low energy data from the new physics parameterized by the nonstandard interaction terms in  $\mathcal{L}$  in Eq. (20) and contributions from the SM without a Higgs boson.



### 3 the Top Quark Couplings to Gauge Bosons

As we discussed in the previous section, one possibility of new physics effects is the modification of the vertices  $b\text{-}b\text{-}Z$ ,  $t\text{-}t\text{-}Z$ , and  $t\text{-}b\text{-}W$  in the effective Lagrangian by the same order of magnitude [11]. In this case, only the vertex  $b\text{-}b\text{-}Z$  can have large contributions to low energy data while, based on the dimensional counting method, the contributions from the other two vertices  $t\text{-}t\text{-}Z$  and  $t\text{-}b\text{-}W$  are suppressed by  $1/16\pi^2$  due to their insertion in loops.

In this case, one can use  $\Gamma_b$  (the partial decay width of the  $Z$  boson to  $\bar{b}b$ ) to constrain the  $b\text{-}b\text{-}Z$  coupling. Denote the nonstandard  $b\text{-}b\text{-}Z$  vertex as

$$\frac{g}{4\cos\theta_W}\kappa\gamma_\mu(1-\gamma_5), \quad (24)$$

which is purely left-handed. In some Extended technicolor models, discussed in Ref. [11], this nonstandard effect arises from the same source as the mass generation of the top quark, therefore  $\kappa$  depends on the top quark mass.

As we will discuss later, the nonuniversal contribution to  $\Gamma_b$  is parameterized by a measurable parameter denoted as  $\epsilon_b$  [14, 15, 16] which is measured to be [14]

$$\epsilon_b(10^3) = 4.4 \pm 7.0. \quad (25)$$

The SM contribution to  $\epsilon_b$  is calculated in Refs. [14, 15], *e.g.*, for a 150 GeV top quark

$$\epsilon_b^{\text{SM}}(10^3) = -4.88. \quad (26)$$

The contribution from  $\kappa$  to  $\epsilon_b$  is

$$\epsilon_b = -\kappa. \quad (27)$$

Within a 95% confidence level (CL), from  $\epsilon_b$  we find that

$$-22.9 \leq \kappa(10^3) \leq 4.4. \quad (28)$$

As an example, the simple commuting extended technicolor model presented in Ref. [11] predicts that

$$\kappa \approx \frac{1}{2}\xi^2 \frac{m_t}{4\pi v}, \quad (29)$$

where  $\xi$  is of order 1. Also in that model the top quark couplings  $\kappa_L^{\text{NC}}$ ,  $\kappa_R^{\text{NC}}$ , and  $\kappa_L^{\text{CC}}$ , as defined in Eqs. (19) and (20), are of the same order as  $\kappa$ . For a 150 GeV top quark, this model predicts

$$\kappa(10^3) \approx 24.3\xi^2. \quad (30)$$

Hence, such a model is likely to be excluded using low energy data.

We will devote the subsequent discussion to models in which the nonstandard  $b\text{-}b\text{-}Z$  coupling can be ignored relative to the  $t\text{-}t\text{-}Z$  and  $t\text{-}b\text{-}W$  couplings. In this case one needs to study their effects at the quantum level, *i.e.*, through loop insertion. We will first discuss the general case where no relations between the couplings are assumed. Later we will impose a relation between  $\kappa_L^{\text{NC}}$  and  $\kappa_L^{\text{CC}}$  which are defined in Eqs. (19) and (20) using an effective model with an approximate custodial symmetry.

### 3.1 General case

The chiral Lagrangian in general has a complicated structure and many arbitrary coefficients which weaken its predictive power. Still, with a few further assumptions, based on the status of present low energy data, the chiral Lagrangian can provide a useful approach to confine the coefficients parameterizing new physics effects.

In this subsection, we provide a general treatment for the case under study with minimal imposed assumptions in the chiral Lagrangian. In this case, we only impose the assumption that the vertex  $b\text{-}b\text{-}Z$  is not modified by the dynamics. In the chiral Lagrangian  $\mathcal{L}$ , as defined in Eqs. (19) and (20), there are six independent parameters ( $\kappa$ 's) which need to be constrained using precision data. Throughout this lecture we will only consider the insertion of  $\kappa$ 's once in one-loop diagrams by assuming that these nonstandard couplings are small;  $\kappa_{L,R}^{NC,CC} < 1$ . At the one-loop level the imaginary parts of the couplings do not contribute to those LEP observables of interest. Thus, hereafter we drop the imaginary pieces from the effective couplings, which reduces the number of relevant parameters to four. Since the bottom quark mass is small relative to the top quark mass, we find that  $\kappa_R^{CC}$  does not contribute to low energy data up to the order  $m_t^2 \ln \Lambda^2$  in the  $m_b \rightarrow 0$  limit. With these observations we conclude that only the three parameters  $\kappa_L^{NC}$ ,  $\kappa_R^{NC}$  and  $\kappa_L^{CC}$  can be constrained.

A systematic approach can be implemented for such an analysis based on the scheme used in Refs. [14, 15, 16], where the radiative corrections can be parameterized by 4 independent parameters, three of those parameters  $\epsilon_1$ ,  $\epsilon_2$ , and  $\epsilon_3$  are proportional to the variables  $T$ ,  $U$  and  $S$  [13], and the fourth one;  $\epsilon_b$  is due to the Glashow-Iliopoulos-Miani- (GIM) violating contribution in  $Z \rightarrow b\bar{b}$  [14].

These parameters are derived from four basic measured observables,  $\Gamma_\ell$  (the partial width of  $Z$  to a charged lepton pair),  $A_{FB}^\ell$  (the forward-backward asymmetry at the  $Z$  peak for the charged lepton  $\ell$ ),  $M_W/M_Z$ , and  $\Gamma_b$  (the partial width of  $Z$  to a  $b\bar{b}$  pair). The expressions of these observables in terms of  $\epsilon$ 's were given in Refs. [14, 15]. In this lecture we only give the relevant terms in  $\epsilon$ 's which might contain the leading effects from new physics.

We denote the vacuum polarization for the  $W^1, W^2, W^3, B$  gauge bosons as

$$\Pi_{\mu\nu}^{ij}(q) = -ig_{\mu\nu} [A^{ij}(0) + q^2 F^{ij}(q^2)] + q_\mu q_\nu \text{ terms}, \quad (31)$$

where  $i, j = 1, 2, 3, 0$  for  $W^1, W^2, W^3$  and  $B$ , respectively. Therefore,

$$\epsilon_1 = e_1 - e_5, \quad (32)$$

$$\epsilon_2 = e_2 - c^2 e_5, \quad (33)$$

$$\epsilon_3 = e_3 - c^2 e_5, \quad (34)$$

$$\epsilon_b = e_b, \quad (35)$$

where

$$e_1 = \frac{A^{33}(0) - A^{11}(0)}{M_W^2}, \quad (36)$$

Figure 1: Some of the relevant Feynman diagrams in the 't Hooft-Feynman gauge, which contribute to the order  $\mathcal{O}(m_t^2 \ln \Lambda^2)$ .

$$e_2 = F^{11}(M_W^2) - F^{33}(M_Z^2), \quad (37)$$

$$e_3 = \frac{c}{s} F^{30}(M_Z^2), \quad (38)$$

$$e_5 = M_Z^2 \frac{dF^{ZZ}}{dq^2}(M_Z^2), \quad (39)$$

and  $c \equiv \cos \theta_W$ .

$$c^2 \equiv \frac{1}{2} \left[ 1 + \left( 1 - \frac{4\pi\alpha(M_Z)}{\sqrt{2}G_f M_Z^2} \right)^{1/2} \right], \quad (40)$$

and  $s^2 = 1 - c^2$ .  $e_b$  is defined through the GIM-violating  $Z \rightarrow b\bar{b}$  vertex

$$V_\mu^{GIM}(Z \rightarrow b\bar{b}) = -\frac{g}{2c} e_b \gamma_\mu \frac{1 - \gamma_5}{2}. \quad (41)$$

$\epsilon_1$  depends quadratically on  $m_t$  [14, 15] and has been measured to good accuracy, therefore  $\epsilon_1$  is sensitive to any new physics coming through the top quark. On the contrary,  $\epsilon_2$  and  $\epsilon_3$  do not play any significant role in our analysis because their dependence on the top mass is only logarithmic.

Non-renormalizability of the effective Lagrangian presents a major issue of how to find a scheme to handle both the divergent and the finite pieces in loop calculations [32, 33]. Such a problem arises because one does not know the underlying theory; hence, no matching can be performed to extract the correct scheme to be used in the effective Lagrangian [17]. One approach is to associate the divergent piece in loop calculations with a physical cutoff  $\Lambda$ , the upper scale at which the effective Lagrangian is valid [27]. In the chiral Lagrangian approach this cutoff  $\Lambda$  is taken to be  $4\pi v \sim 3 \text{ TeV}$  [17].<sup>2</sup> For the finite piece no completely satisfactory approach is available [32].

To perform calculations using the chiral Lagrangian, one should arrange the contributions in powers of  $1/4\pi v$  and then include all diagrams up to the desired power. In the  $R_\xi$  gauge ( $\Sigma \neq 1$ ), the couplings of the Goldstone bosons to the fermions should also be included in Feynman diagram calculations. These couplings can be easily found by expanding the terms in  $\mathcal{L}$  as given in Eq. (19). We will not give the explicit expressions for those terms here. Some of the relevant Feynman diagrams are shown in Fig. 1. Calculations were done in the 't Hooft-Feynman gauge. We have also checked our calculations in both the Landau gauge and the unitary gauge and found agreement as expected.

We calculate the contribution to  $\epsilon_1$  and  $\epsilon_b$  due to the new interaction terms in the chiral Lagrangian (see Eqs. (19) and (20)) using the dimensional regularization scheme

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<sup>2</sup> This scale,  $4\pi v \sim 3 \text{ TeV}$ , is only meant to indicate the typical cutoff scale. It is equally probable to have, say,  $\Lambda = 1 \text{ TeV}$ .

and taking the bottom mass to be zero. At the end of the calculation, we replace the divergent piece  $1/\epsilon$  by  $\ln(\Lambda^2/m_t^2)$  for  $\epsilon = (4 - n)/2$  where  $n$  is the space-time dimension. We have assumed that the underlying full theory is renormalizable. The cutoff scale  $\Lambda$  serves as the infrared cutoff of the operators in the effective Lagrangian. Due to the renormalizability of the full theory, from renormalization group analysis, we conclude that the same cutoff  $\Lambda$  should also serve as the ultraviolet cutoff of the effective Lagrangian in calculating Wilson coefficients. Hence, in the dimensional regularization scheme,  $1/\epsilon$  is replaced by  $\ln(\Lambda^2/\mu^2)$ . Furthermore, the renormalization scale  $\mu$  is set to be  $m_t$ , the heaviest mass scale in the effective Lagrangian of interest.

Since we are mainly interested in new physics associated with the top quark couplings to gauge bosons, we shall restrict ourselves to the *leading* contribution enhanced by the top quark mass, *i.e.*, of the order of  $m_t^2 \ln \Lambda^2$ . We find

$$\epsilon_1 = \frac{G_F}{2\sqrt{2}\pi^2} 3m_t^2 (-\kappa_L^{\text{NC}} + \kappa_R^{\text{NC}} + \kappa_L^{\text{CC}}) \ln \frac{\Lambda^2}{m_t^2}, \quad (42)$$

$$\epsilon_b = \frac{G_F}{2\sqrt{2}\pi^2} m_t^2 \left( -\frac{1}{4} \kappa_R^{\text{NC}} + \kappa_L^{\text{NC}} \right) \ln \frac{\Lambda^2}{m_t^2}. \quad (43)$$

Note that  $\epsilon_2$  and  $\epsilon_3$  do not contribute at this order. That  $\kappa_L^{\text{CC}}$  does not contribute to  $\epsilon_b$  up to this order can be understood from Eq. (20). If  $\kappa_L^{\text{CC}} = -1$  then there is no net  $t$ - $b$ - $W$  coupling in the chiral Lagrangian after including both the standard and nonstandard contributions. Hence, no dependence on the top quark mass can be generated, *i.e.*, the non-standard  $\kappa_L^{\text{CC}}$  contribution to  $\epsilon_b$  must cancel the SM contribution when  $\kappa_L^{\text{CC}} = -1$ , independently of the couplings of the neutral current. From this observation and because the SM contribution to  $\epsilon_b$  is finite, we conclude that  $\kappa_L^{\text{CC}}$  cannot contribute to  $\epsilon_b$  at the order of interest.

Note that we set the renormalization scale  $\mu$  to be  $m_t$ , which is the natural scale to be used in our study because the top quark is considered to be the heaviest mass scale in the effective Lagrangian. We have assumed that all other heavy fields have been integrated out to modify the effective couplings of the top quark to gauge bosons at the scale  $m_t$  in the chiral Lagrangian. Here we ignore the effect of the running couplings from the top quark mass scale down to the  $Z$  boson mass scale which is a reasonable approximation for our study.

To constrain these nonstandard couplings we need to have both the experimental values<sup>3</sup> and the SM predictions of  $\epsilon$ 's. First, we tabulate the numerical inputs, taken from Ref. [14], used in our analysis:

$$\begin{aligned} \alpha^{-1}(M_Z^2) &= 128.87 \pm 0.12, \\ G_F &= 1.16637(2) \times 10^{-5} \text{ GeV}^{-2}, \\ M_Z &= 91.187 \pm 0.007 \text{ GeV}, \end{aligned}$$

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<sup>3</sup> We should first discuss the old (1993) data in this lecture, then discuss the impact of new (1994) data in the next lecture for comparison. This is useful to test the sensitivity of the constraints on  $\kappa$ 's from the measurements of  $\epsilon$ 's.

$$\begin{aligned}
M_W/M_Z &= 0.8798 \pm 0.0028, \\
\Gamma_\ell &= 83.52 \pm 0.28 \text{ MeV}, \\
\Gamma_b &= 383 \pm 6 \text{ MeV}, \\
A_{FB}^\ell &= 0.0164 \pm 0.0021, \\
A_{FB}^b &= 0.098 \pm 0.009, \\
A_{LR}(\text{SLC}) &= 0.100 \pm 0.044.
\end{aligned}$$

These values yield [14]

$$\begin{aligned}
\epsilon_1 10^3 &= -0.3 \pm 3.4, \\
\epsilon_b 10^3 &= 4.4 \pm 7.0,
\end{aligned}$$

and, for completeness,

$$\begin{aligned}
\epsilon_2 10^3 &= -7.6 \pm 7.6, \\
\epsilon_3 10^3 &= 0.4 \pm 4.2.
\end{aligned}$$

The SM contribution to  $\epsilon$ 's have been calculated in Refs. [14, 15]. We will include these contributions in our analysis in accordance with the assumed Higgs boson mass. In the light Higgs boson case ( $m_H < m_t$ ), the calculated values of the  $\epsilon$ 's include both the SM contribution calculated in Refs. [14, 15] and the new physics contribution derived from the effective couplings of the top quark to gauge bosons. In the heavy Higgs boson case ( $m_H > m_t$ ) we subtract the Higgs boson contribution from the SM calculations of  $\epsilon$ 's given in Refs. [14, 15]. In this case, the Higgs boson contribution is implicitly included in the effective couplings of the top quark to gauge bosons after the heavy Higgs boson field is integrated out. Finally, in a spontaneous symmetry scenario without a Higgs boson the calculations of  $\epsilon$ 's are exactly the same as those done in the heavy Higgs boson case except that the effective couplings of the top quark to gauge bosons are not due to an assumed heavy Higgs boson in the full theory.

Choosing  $m_t = 150 \text{ GeV}$  and  $m_H = 100 \text{ GeV}$  we span the parameter space defined by  $-1 \leq \kappa_L^{\text{NC}} \leq 1$ ,  $-1 \leq \kappa_R^{\text{NC}} \leq 1$ , and  $-1 \leq \kappa_L^{\text{CC}} \leq 1$ . Within 95% CL and including both the SM and the new physics contributions, the allowed region of these three parameters is found to form a thin slice in the specified volume. The two-dimensional projections of this slice were shown in Figs. 2, 3, and 4 of Ref. [34].<sup>4</sup> These nonstandard couplings ( $\kappa$ 's) do exhibit some interesting features.

- (1) As a function of the top quark mass, the allowed volume for the top quark couplings to gauge bosons shrinks as the top quark becomes more massive.
- (2) New physics prefers positive  $\kappa_L^{\text{NC}}$ .  $\kappa_L^{\text{NC}}$  is constrained within  $-0.3$  to  $0.6$  ( $-0.2$  to  $0.5$ ) for a  $150$  ( $175$ )  $\text{GeV}$  top quark.

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<sup>4</sup> We do not reproduce those figures here because they exhibit the same shape as those obtained using new data, to be discussed in the next lecture.

(3) New physics prefers  $\kappa_L^{\text{CC}} \approx -\kappa_R^{\text{NC}}$ .

A similar analysis was carried out in Ref. [10], in which, however, the authors did not include the charged current contribution and assumed only the vertex  $t$ - $t$ - $Z$  gives large nonstandard effects. The allowed region they found simply corresponds, in our analysis, to the region defined by the intersection of the allowed volume and the plane  $\kappa_L^{\text{CC}} = 0$ . This gives a small area confined in the vicinity of the line  $\kappa_L^{\text{NC}} = \kappa_R^{\text{NC}}$ . This can be understood from the expression of  $\epsilon_1$  derived in Eq. (42). After setting  $\kappa_L^{\text{CC}} = 0$ , we find

$$\epsilon_1 \propto (\kappa_R^{\text{NC}} - \kappa_L^{\text{NC}}) . \quad (44)$$

In this case we note that the length of the allowed area is merely determined by the contribution from  $\epsilon_b$ . We will elaborate on a more quantitative comparison in the second part of this section.

### 3.2 *Special case*

The allowed region in the parameter space obtained in Figs. 2, 3 and 4 of Ref. [34] contains all possible new physics (to the order  $m_t^2 \ln \Lambda^2$ ) which can modify the couplings of the top quark to gauge bosons as described by  $\kappa_L^{\text{NC}}$ ,  $\kappa_R^{\text{NC}}$ , and  $\kappa_L^{\text{CC}}$ . In this section we would like to examine a special class of models in which an approximate custodial symmetry is assumed as suggested by low energy data.

The SM has an additional (accidental) symmetry called the custodial symmetry which is responsible for the tree-level relation:  $\rho = 1$ . This symmetry is slightly broken at the quantum level by the  $SU(2)_L$  doublet fermion mass splitting and the hypercharge coupling  $g'$  [35]. Writing  $\rho = 1 + \delta\rho$ ,  $\delta\rho$  would vanish to all orders if this symmetry is exact. Because low energy data indicate that  $\delta\rho$  is very close to zero we shall therefore assume an underlying theory with a custodial symmetry. In other words we require the global group  $SU(2)_V$  associated with the custodial symmetry to be a subgroup of the full group characterizing the full theory. We will assume that the custodial symmetry is broken by the same factors which break it in the SM, *i.e.*, by the fermion mass splitting and the hypercharge coupling  $g'$ .

In the chiral Lagrangian this assumption of a custodial symmetry sets  $v_3 = v$ , and forces the couplings of the top quark to gauge bosons  $W_\mu^a$  to be equal after turning off the hypercharge and assuming  $m_b = m_t$ . If the dynamics of the symmetry-breaking is such that the masses of the two  $SU(2)$  partners  $t$  and  $b$  remain degenerate then we expect new physics to contribute to the couplings of  $t$ - $t$ - $Z$  and  $t$ - $b$ - $W$  by the same amount. However, in reality,  $m_b \ll m_t$ ; thus, the custodial symmetry has to be broken. We will discuss how this symmetry is broken shortly. Since we are mainly interested in the *leading* contribution enhanced by the top quark mass at the order  $m_t^2 \ln \Lambda^2$ , turning the hypercharge coupling on and off will not affect the final result up to this order.

We can construct the two Hermitian operators  $J_L$  and  $J_R$ , which transform under  $G$  as

$$J_L^\mu = -i\Sigma D_\mu \Sigma^\dagger \rightarrow g_L J_L^\mu g_L^\dagger , \quad (45)$$

$$J_R^\mu = i\Sigma^\dagger D_\mu \Sigma \rightarrow g_R J_R^\mu g_R^\dagger, \quad (46)$$

where  $g_L = e^{i\alpha^a \frac{\tau^a}{2}} \in SU(2)_L$  and  $g_R = e^{iy \frac{\tau^3}{2}}$  (note that  $v_3 = v$  in  $\Sigma$ ). In fact, using either  $J_L$  or  $J_R$  will lead to the same result. Hence, from now on we will only consider  $J_R$ . The SM Lagrangian can be derived from<sup>5</sup>

$$\begin{aligned} \mathcal{L}_0 = & \overline{\Psi}_L i\gamma^\mu D_\mu^L \Psi_L + \overline{\Psi}_R i\gamma^\mu D_\mu^R \Psi_R - (\overline{\Psi}_L \Sigma M \Psi_R + h.c.) \\ & - \frac{1}{4} W_{\mu\nu}^a W^{\mu\nu a} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + \frac{v^2}{4} \text{Tr}(J_R^\mu J_{R\mu}), \end{aligned} \quad (47)$$

where  $M$  is a diagonal mass matrix. We have chosen the left-handed fermion fields to be the ones defined in Eq. (7):

$$\Psi_L \equiv \Sigma \begin{pmatrix} t \\ b \end{pmatrix}_L. \quad (48)$$

The right-handed fermion fields  $t_R$  and  $b_R$  coincide with the original right-handed fields (see Eq. (9)). Also

$$D_\mu^L = \partial_\mu - ig W_\mu^a \frac{\tau^a}{2} - ig' B_\mu \frac{Y}{2}, \quad (49)$$

$$D_\mu^R = \partial_\mu - ig' B_\mu \left( \frac{Y}{2} + \frac{\tau^3}{2} \right). \quad (50)$$

Note that in the nonlinear realized effective theories using either set of fields ( $\Psi_{L,R}$  or  $F_{L,R}$ ) to construct a chiral Lagrangian will lead to the same S matrix [24].

The Lagrangian  $\mathcal{L}_0$  in Eq. (47) is not the most general Lagrangian one can construct based solely on the symmetry of  $G/H$ . Taking advantage of the chiral Lagrangian approach we can derive additional interaction terms which deviate from the SM. This is so because in this formalism the  $SU(2)_L \times U(1)_Y$  symmetry is nonlinearly realized and only the  $U(1)_{em}$  is linearly realized.

Because the SM is so successful one can think of the SM (without the top quark) as being the leading term in the expansion of the effective Lagrangian. Any possible deviation associated with the light fields can only come through higher dimensional operators in the Lagrangian. However, this assumption is neither necessary nor preferable when dealing with the top quark because no precise data are available to lead to such a conclusion. In this lecture we will include nonstandard dimension-four operators for the couplings of the top quark to gauge bosons. In fact this is all we will deal with and we will not consider operators with dimension higher than four. Note that higher dimensional operators are naturally suppressed by powers of  $1/\Lambda$ .

One can write  $J_R$  as

$$J_R^\mu = J_R^{\mu a} \frac{\tau^a}{2}, \quad (51)$$

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<sup>5</sup> This Lagrangian is equivalent to the one defined in Eq. (17). Both of them give the same S-matrix for any physical process.

with

$$J_R^{\mu a} = \text{Tr}(\tau^a J_R^\mu) = i \text{Tr}(\tau^a \Sigma^\dagger D^\mu \Sigma) . \quad (52)$$

The full operator  $J_R$  posses an explicit custodial symmetry when  $g' = 0$  as can easily be checked by expanding it in powers of the Goldstone boson fields.

Consider first the left-handed sector. One can add additional interaction terms to the Lagrangian  $\mathcal{L}_0$

$$\mathcal{L}_1 = \kappa_1 \overline{\Psi}_L \gamma_\mu \Sigma J_R^\mu \Sigma^\dagger \Psi_L + \kappa_2 \overline{\Psi}_L \gamma_\mu \Sigma \tau^3 J_R^\mu \Sigma^\dagger \Psi_L + \kappa_2^\dagger \overline{\Psi}_L \gamma_\mu \Sigma J_R^\mu \tau^3 \Sigma^\dagger \Psi_L , \quad (53)$$

where  $\kappa_1$  is an arbitrary real parameter and  $\kappa_2$  is an arbitrary complex parameter. Here we do not include interaction terms such as

$$\kappa_3 \overline{\Psi}_L \gamma_\mu \Sigma \tau^3 J_R^\mu \tau^3 \Sigma^\dagger \Psi_L , \quad (54)$$

where  $\kappa_3$  is real, because it is simply a linear combination of the other two terms in  $\mathcal{L}_1$ . This can be easily checked by using Eq. (51) and the commutation relations of the Pauli matrices. Note that  $\mathcal{L}_1$  still is not the most general Lagrangian one can write for the left-handed sector, as compared to Eq. (19). In fact, it is our insistence on using the fermion doublet form and the full operator  $J_R$  that lead us to this form. For shorthand,  $\mathcal{L}_1$  can be further rewritten as

$$\mathcal{L}_1 = \overline{\Psi}_L \gamma_\mu \Sigma K_L J_R^\mu \Sigma^\dagger \Psi_L + \overline{\Psi}_L \gamma_\mu \Sigma J_R^\mu K_L^\dagger \Sigma^\dagger \Psi_L , \quad (55)$$

where  $K_L$  is a complex diagonal matrix.

These new terms can be generated either through some electroweak symmetry-breaking scenario or through some other new heavy physics effects. If  $m_b = m_t$  and  $g' = 0$ , then we require the effective Lagrangian to respect fully the custodial symmetry to all orders. In this limit,  $\kappa_2 = 0$  in Eq. (53) and  $K_L = \kappa_1 \mathbf{1}$ , where  $\mathbf{1}$  is the unit matrix and  $\kappa_1$  is real.

Since  $m_b \ll m_t$ , we can think of  $\kappa_2$  as generated through the evolution from  $m_b = m_t$  to  $m_b = 0$ . In the matrix notation this implies  $K_L$  is not proportional to the unit matrix and can be parameterized by

$$K_L = \begin{pmatrix} \kappa_L^t & 0 \\ 0 & \kappa_L^b \end{pmatrix} , \quad (56)$$

with

$$\kappa_L^t = \frac{\kappa_1}{2} + \kappa_2 , \quad (57)$$

and

$$\kappa_L^b = \frac{\kappa_1}{2} - \kappa_2 . \quad (58)$$

In the unitary gauge we get the terms

$$\begin{aligned} & + \frac{g}{2c} 2\text{Re}(\kappa_L^t) \overline{t}_L \gamma^\mu t_L Z_\mu + \frac{g}{\sqrt{2}} (\kappa_L^t + \kappa_L^{b\dagger}) \overline{t}_L \gamma^\mu b_L W_\mu^+ \\ & + \frac{g}{\sqrt{2}} (\kappa_L^b + \kappa_L^{t\dagger}) \overline{b}_L \gamma^\mu t_L W_\mu^- - \frac{g}{2c} 2\text{Re}(\kappa_L^b) \overline{b}_L \gamma^\mu b_L Z_\mu . \end{aligned} \quad (59)$$



As discussed in the previous section, we will assume that new physics effects will not modify the  $b_L$ - $b_L$ - $Z$  vertex. This can be achieved by choosing  $\kappa_1 = 2\text{Re}(\kappa_2)$  such that  $\text{Re}(\kappa_L^b)$  vanishes in Eq. (58). Later, in Sec. 4, we will consider a specific model to support this assumption.

Since the imaginary parts of the couplings do not contribute to LEP physics of interest, we simply drop them hereafter. With this assumption we are left with one real parameter  $\kappa_L^t$  which will be denoted from now on by  $\kappa_L/2$ . The left-handed top quark couplings to the gauge bosons are

$$t_L - t_L - Z : \frac{g}{4c} \kappa_L \gamma_\mu (1 - \gamma_5), \quad (60)$$

$$t_L - b_L - W : \frac{g}{2\sqrt{2}} \frac{\kappa_L}{2} \gamma_\mu (1 - \gamma_5). \quad (61)$$

Notice the connection between the neutral and the charged current, as compared to Eq. (20):

$$\kappa_L^{\text{NC}} = 2\kappa_L^{\text{CC}} = \kappa_L. \quad (62)$$

This conclusion holds for any underlying theory with an approximate custodial symmetry such that the vertex  $b$ - $b$ - $Z$  is not modified as discussed above.

For the right-handed sector, the situation is different because the right-handed fermion fields are  $SU(2)$  singlet, hence the induced interactions do not see the full operator  $J_R$  but its components individually. Therefore, we cannot impose the previous connection between the neutral and charged current couplings.

The additional allowed interaction terms in the right-handed sector are given by

$$\begin{aligned} \mathcal{L}_2 = & \frac{g}{2c} \kappa_R^t{}^{\text{NC}} \overline{t_R} \gamma^\mu t_R J_{R\mu}^3 + \frac{g}{\sqrt{2}} \kappa_R^{\text{CC}} \overline{t_R} \gamma^\mu b_R J_{R\mu}^+ \\ & + \frac{g}{\sqrt{2}} \kappa_R^{\text{CC}\dagger} \overline{b_R} \gamma^\mu t_R J_{R\mu}^- - \frac{g}{2c} \kappa_R^b{}^{\text{NC}} \overline{b_R} \gamma^\mu b_R J_{R\mu}^3, \end{aligned} \quad (63)$$

where  $\kappa_R^t{}^{\text{NC}}$  and  $\kappa_R^b{}^{\text{NC}}$  are two arbitrary real parameters and  $\kappa_R^{\text{CC}}$  is an arbitrary complex parameter. Note that in  $\mathcal{L}_2$  we have one more additional coefficient than we have in  $\mathcal{L}_1$  (in Eq. (53)), this is due to our previous assumption of using the full operator  $J_R$  in constructing the left-handed interactions. We assume that the  $b_R$ - $b_R$ - $Z$  vertex just as the  $b_L$ - $b_L$ - $Z$  vertex is not modified, then the coefficient  $\kappa_R^b{}^{\text{NC}}$  vanishes. Because  $\kappa_R^{\text{CC}}$  does not contribute to LEP physics in the limit of  $m_b = 0$  and at the order  $m_t^2 \ln \Lambda^2$  we are left with one real parameter  $\kappa_R^t{}^{\text{NC}}$  which will be denoted hereafter as  $\kappa_R$ . The right-handed top quark coupling to  $Z$  boson is

$$t_R - t_R - Z : \frac{g}{4c} \kappa_R \gamma_\mu (1 + \gamma_5). \quad (64)$$

In the rest of this section we consider models described by  $\mathcal{L}_1$  and  $\mathcal{L}_2$  with only two relevant parameters  $\kappa_L$  and  $\kappa_R$ . Performing the calculations as we discussed in the previous subsection we find

$$\epsilon_1 = \frac{G_F}{2\sqrt{2}\pi^2} 3m_t^2 \left( \kappa_R - \frac{\kappa_L}{2} \right) \ln\left(\frac{\Lambda^2}{m_t^2}\right), \quad (65)$$

$$\epsilon_b = \frac{G_F}{2\sqrt{2}\pi^2} m_t^2 \left( -\frac{1}{4}\kappa_R + \kappa_L \right) \ln\left(\frac{\Lambda^2}{m_t^2}\right). \quad (66)$$

These results simply correspond to those in Eqs. (42) and (43) after substituting  $\kappa_L^{\text{NC}} = 2\kappa_L^{\text{CC}} = \kappa_L$  and  $\kappa_R^{\text{NC}} = \kappa_R$ .

The constraints on  $\kappa_L$  and  $\kappa_R$  for models with a light Higgs boson or a heavy Higgs boson, or without a physical scalar (such as a Higgs boson) are presented here in order. Let us first consider a standard light Higgs boson with mass  $m_H = 100$  GeV. Including the SM contribution from Ref. [14] we span the plane defined by  $\kappa_L$  and  $\kappa_R$  for top mass 150 and 175 GeV, respectively. Figs. 5 and 6 of Ref. [34] showed the allowed range for those parameters within 95% CL. As a general feature one observes that the allowed range is a narrow area aligned close to the line  $\kappa_L = 2\kappa_R$  where for  $m_t = 150$  GeV the maximum range for  $\kappa_L$  is between  $-0.1$  and  $0.5$ . As the top mass increases this range shrinks and moves downward and to the right away from the origin  $(\kappa_L, \kappa_R) = (0, 0)$ . The deviation from the relation  $\kappa_L = 2\kappa_R$  for various top quark masses was given in Fig. 7 of Ref. [34] by calculating  $\kappa_L - 2\kappa_R$  as a function of  $m_t$ . Note that the SM has the solution  $\kappa_L = \kappa_R = 0$ . This solution ceases to exist for  $m_t \geq 200$  GeV. The special relation  $\kappa_L = 2\kappa_R$  is a consequence of the assumption we imposed in connecting the left-handed neutral and charged current.

It is worth mentioning that the SM contribution to  $\epsilon_b$  is lower than the experimental central value [14, 15]. This is reflected in the behavior of  $\kappa_L$  which prefers being positive to compensate this difference as can be seen from Eq. (66). This means in models of electroweak symmetry-breaking with an approximate custodial symmetry, a positive  $\kappa_L$  is preferred. In Fig. 8 of Ref. [34] we showed the allowed  $\kappa_L^{\text{CC}} = \kappa_L^{\text{NC}}/2 = \kappa_L/2$  as a function of  $m_t$ . With new physics effects ( $\kappa_L \neq 0$ )  $m_t$  can be as large as 300 GeV, although in the SM ( $\kappa_L = 0$ )  $m_t$  is bounded below 200 GeV.

Now, we would like to discuss the effect of a light SM Higgs boson ( $m_H < m_t$ ) on the allowed range of these parameters. It is easy to anticipate the effect; since  $\epsilon_b$  is not sensitive to the Higgs boson contribution up to one loop [14], the allowed range is only affected by the Higgs boson contribution to  $\epsilon_1$  which affects slightly the width of the allowed area and its location relative to the line  $\kappa_L = 2\kappa_R$ . One expects that as the Higgs boson mass increases the allowed area moves upward. The reason simply lies in the fact that the standard Higgs boson contribution to  $\epsilon_1$  up to one loop becomes more negative for heavier Higgs boson, hence  $2\kappa_R$  prefers to be larger than  $\kappa_L$  to compensate this effect. However, this modification is not significant because  $\epsilon_1$  depends on the Higgs boson mass only logarithmically [15].

If there is a heavy Higgs boson ( $m_H > m_t$ ), then it should be integrated out from the full theory and its effect in the chiral Lagrangian is manifested through the effective couplings of the top quark to gauge bosons. In this case we simply subtract the Higgs boson contribution from the SM results obtained in Refs. [14, 15]. Fig. 9 of Ref. [34] showed the allowed area in the  $\kappa_L$  and  $\kappa_R$  plane for a 175 GeV top quark in such models. Again we find no noticeable difference between the results from these models and those with a light Higgs boson. That is because up to one loop level neither  $\epsilon_1$  nor  $\epsilon_b$  is sensitive to the Higgs boson contribution [14, 15].

Figure 2: A comparison between our model and the model in Ref. 7. The allowed regions in both models are shown on the plane of  $\kappa_L^{\text{NC}}$  and  $\kappa_R^{\text{NC}}$ , for  $m_t = 150 \text{ GeV}$ .

If we consider a new symmetry-breaking scenario without a fundamental scalar such as a SM Higgs boson, following the previous discussions we again find negligible effects on the allowed range of  $\kappa_L$  and  $\kappa_R$ .

What we learned is that to infer a bound on the Higgs boson mass from the measurement of the effective couplings of the top quark to gauge bosons, we need very precise measurement of the parameters  $\kappa_L$  and  $\kappa_R$ . However, from the correlations between the effective couplings ( $\kappa$ 's) of the top quark to gauge bosons, we can infer if the symmetry-breaking sector is due to a Higgs boson or not, *i.e.*, we may be able to probe the symmetry-breaking mechanism in the top quark system. Further discussion will be given in the next section.

Finally, we would like to compare our results with those in Ref. [10]. Fig. 2 shows the most general allowed region for the couplings  $\kappa_L^{\text{NC}}$  and  $\kappa_R^{\text{NC}}$ , *i.e.*, without imposing any relation between  $\kappa_L^{\text{NC}}$  and  $\kappa_L^{\text{CC}}$ . This region is for top mass 150 GeV and is covering the parameter space  $-1.0 \leq \kappa_L^{\text{NC}}, \kappa_R^{\text{NC}} \leq 1.0$ . We find

$$\begin{aligned} -0.3 &\leq \kappa_L^{\text{NC}} \leq 0.6, \\ -1.0 &\leq \kappa_R^{\text{NC}} \leq 1.0. \end{aligned}$$

Also shown on Fig. 2 the allowed regions from our model and the model in Ref. [10]. The two regions overlap in the vicinity of the origin (0, 0) which corresponds to the SM case. As  $\kappa_L^{\text{NC}} \geq 0.1$ , these two regions diverge and become separable. One notices that the allowed range predicted in Ref. [10] lies along the line  $\kappa_L^{\text{NC}} = \kappa_R^{\text{NC}}$  whereas in our case the slope is different  $\kappa_L^{\text{NC}} = 2\kappa_R^{\text{NC}}$ . This difference comes in because of the assumed dependence of  $\kappa_L^{\text{CC}}$  on the other two couplings  $\kappa_L^{\text{NC}}$  and  $\kappa_R^{\text{NC}}$ . In our case  $\kappa_L^{\text{CC}} = \kappa_L^{\text{NC}}/2$ , and in Ref. [10]  $\kappa_L^{\text{CC}} = 0$ .

Note that for  $m_t \leq 200 \text{ GeV}$  the allowed region of  $\kappa$ 's in all models of symmetry-breaking should overlap near the origin because the SM is consistent with low energy data at the 95% CL. If we imagine that any prescribed dependence between the couplings corresponds to a symmetry-breaking scenario, then, given the present status of low energy data, it is possible to distinguish between different scenarios if  $\kappa_L^{\text{NC}}$ ,  $\kappa_R^{\text{NC}}$  and  $\kappa_L^{\text{CC}}$  are larger than 10%. Better future measurements of  $\epsilon$ 's can further discriminate between different symmetry-breaking scenarios. We will discuss how the SLC and the NLC can contribute to these measurements in Sec. 5, and defer the discussions on hadron colliders (such as the Tevatron and the LHC) in the third lecture. Before that, let us examine a specific model that predicts certain relations among the coefficients  $\kappa_L^{\text{CC}}$ ,  $\kappa_R^{\text{CC}}$ ,  $\kappa_L^{\text{NC}}$  and  $\kappa_R^{\text{NC}}$  of the effective couplings of the top quark to gauge bosons.

## 4 Heavy Higgs Limit in the SM

The goal of this study is to probe new physics effects, particularly the effects due to the symmetry-breaking sector, in the top quark system by examining the couplings of top quark to gauge bosons. To illustrate how a specific symmetry-breaking mechanism might affect these couplings, we consider in this section the Standard Model with a heavy Higgs boson ( $m_H > m_t$ ) as the full theory, and derive the effective couplings  $\kappa_L^{\text{NC}}$ ,  $\kappa_R^{\text{NC}}$ ,  $\kappa_L^{\text{CC}}$ , and  $\kappa_R^{\text{CC}}$  at the top quark mass scale in the effective Lagrangian after integrating out the heavy Higgs boson field.

Given the full theory (SM in this case), we can perform matching between the underlying theory and the effective Lagrangian. In this case, the heavy Higgs boson mass acts as a regulator (cutoff) of the effective theory[36].

While setting  $m_b = 0$ , and only keeping the leading terms of the order  $m_t^2 \ln m_H^2$ , we find the following effective couplings

$$t - t - Z : \frac{g}{4c} \frac{G_F}{2\sqrt{2}\pi^2} \left( \frac{-1}{8} m_t^2 \gamma_\mu (1 - \gamma_5) + \frac{1}{8} m_t^2 \gamma_\mu (1 + \gamma_5) \right) \ln \left( \frac{m_H^2}{m_t^2} \right), \quad (67)$$

$$t - b - W : \frac{g}{2\sqrt{2}} \frac{G_F}{2\sqrt{2}\pi^2} \left( \frac{-1}{16} \right) m_t^2 \gamma_\mu (1 - \gamma_5) \ln \left( \frac{m_H^2}{m_t^2} \right). \quad (68)$$

From this we conclude

$$\kappa_L^{\text{NC}} = 2\kappa_L^{\text{CC}} = \frac{G_F}{2\sqrt{2}\pi^2} \left( \frac{-1}{8} \right) m_t^2 \ln \left( \frac{m_H^2}{m_t^2} \right), \quad (69)$$

$$\kappa_R^{\text{NC}} = \frac{G_F}{2\sqrt{2}\pi^2} \frac{1}{8} m_t^2 \ln \left( \frac{m_H^2}{m_t^2} \right), \quad (70)$$

$$\kappa_R^{\text{CC}} = 0. \quad (71)$$

Note that the relation between the left-handed currents ( $\kappa_L^{\text{NC}} = 2\kappa_L^{\text{CC}}$ ) agree with our prediction because of the approximate custodial symmetry in the full theory (SM) and the fact that vertex  $b-b-Z$  is not modified. The right-handed currents  $\kappa_R^{\text{CC}}$  and  $\kappa_R^{\text{NC}}$  are not correlated, and  $\kappa_R^{\text{CC}}$  vanishes for a massless  $b$ . Also note an additional relation in the effective Lagrangian between the left- and right-handed effective couplings of the top quark to  $Z$  boson, *i.e.*,

$$\kappa_L^{\text{NC}} = -\kappa_R^{\text{NC}}. \quad (72)$$

This means only the axial vector current of  $t-t-Z$  acquires a nonuniversal contribution but its vector current is not modified.

As discussed in Sec. 2, due to the Ward identities associated with the photon field there can be no nonuniversal contribution to either the  $b-b-A$  or  $t-t-A$  vertex after renormalizing the fine structure constant  $\alpha$ . This can be explicitly checked in this model. Furthermore, up to the order of  $m_t^2 \ln m_H^2$ , the vertex  $b-b-Z$  is not modified which agrees with the assumption we made in Sec. 2 that there exist dynamics of

electroweak symmetry-breaking so that neither  $b_R$ - $b_R$ - $Z$  nor  $b_L$ - $b_L$ - $Z$  in the effective Lagrangian is modified at the scale of  $m_t$ .

From this example we learn that the effective couplings of the top quark to gauge bosons arising from a heavy Higgs boson are correlated in a specific way: namely,

$$\kappa_L^{\text{NC}} = 2\kappa_L^{\text{CC}} = -\kappa_R^{\text{NC}}. \quad (73)$$

(This relation in general also holds for models with a heavy scalar which is not necessarily a SM Higgs boson, *i.e.*, the coefficients of the last two terms in Eq. (22) can be arbitrary, and are not necessarily 1/2 and 1/4, respectively). In other words, if the couplings of a heavy top quark to the gauge bosons are measured and exhibit large deviations from these relations, then it is likely that the electroweak symmetry-breaking is not due to the standard Higgs mechanism which contains a heavy SM Higgs boson. This illustrates how the symmetry-breaking sector can be probed by measuring the effective couplings of the top quark to gauge bosons.

## 5 Direct Measurement of the Top Quark Couplings

In Sec. 3 we concluded that the precision LEP data can constrain the couplings  $\kappa_L^{\text{NC}}$ ,  $\kappa_R^{\text{NC}}$  and  $\kappa_L^{\text{CC}}$ , but not  $\kappa_R^{\text{CC}}$  (the right-handed charged current). In this section we examine how to improve our knowledge on these couplings at the current and future electron colliders. (We defer the discussions on hadron colliders in Lecture Three.)

### 5.1 At the SLC

The measurement of the left-right cross section asymmetry  $A_{LR}$  in  $Z$  production with a longitudinally polarized electron beam at the SLC provides a stringent test of the SM and is sensitive to new physics.

Additional constraints on the couplings  $\kappa_L^{\text{NC}}$ ,  $\kappa_R^{\text{NC}}$  and  $\kappa_L^{\text{CC}}$  can be inferred from  $A_{LR}$  which can be written as [14]

$$A_{LR} = \frac{2x}{1+x^2}, \quad (74)$$

with

$$x = 1 - 4s^2(1 + \Delta k'), \quad (75)$$

$$\Delta k' = \frac{\epsilon_3 - c^2\epsilon_1}{c^2 - s^2}. \quad (76)$$

Up to the order  $m_t^2 \ln \Lambda^2$ , only  $\epsilon_1$  contributes. In our model with the approximate custodial symmetry, *i.e.*,  $\kappa_L^{\text{NC}} = 2\kappa_L^{\text{CC}} = \kappa_L$ , the SLC  $A_{LR}$  measurement will have a significant influence on the precise measurement of the nonuniversal couplings of the top quark. This will decrease the width of the allowed area in the parameter space of  $\kappa_L$  versus  $\kappa_R$ . However, SLC data will have no effect on the length of the allowed

region which in our approximation is solely determined by  $\epsilon_b$ . Hence, a more accurate measurement of  $\epsilon_b$ , *i.e.*,  $\Gamma(Z \rightarrow b\bar{b})$ , is required to further confine the nonuniversal interactions of the top quark to gauge bosons to probe new physics. (More details will be given in the next lecture.)

## 5.2 At the NLC

The best place to probe  $\kappa_L^{\text{NC}}$  and  $\kappa_R^{\text{NC}}$  associated with the  $t$ - $t$ - $Z$  coupling is at the NLC through  $e^-e^+ \rightarrow A, Z \rightarrow t\bar{t}$ . (We use NLC to represent a generic  $e^-e^+$  supercollider [26].) A detailed Monte Carlo study on the measurement of these couplings at the NLC including detector effects and initial state radiation can be found in Ref. [37]. The bounds were obtained by studying the angular distribution and the polarization of the top quark produced in  $e^-e^+$  collisions. Assuming a  $50 \text{ fb}^{-1}$  luminosity at  $\sqrt{S} = 500 \text{ GeV}$ , we concluded that within a 90% confidence level, it should be possible to measure  $\kappa_L^{\text{NC}}$  to within about 8%, while  $\kappa_R^{\text{NC}}$  can be known to within about 18%. A 1 TeV machine can do better than a 500 GeV machine in determining  $\kappa_L^{\text{NC}}$  and  $\kappa_R^{\text{NC}}$  because the relative sizes of the  $t_R(\bar{t})_R$  and  $t_L(\bar{t})_L$  production rates become small and the polarization of the  $t\bar{t}$  pair is purer. Namely, it is more likely to produce either a  $t_L(\bar{t})_R$  or a  $t_R(\bar{t})_L$  pair. A purer polarization of the  $t\bar{t}$  pair makes  $\kappa_L^{\text{NC}}$  and  $\kappa_R^{\text{NC}}$  better determined. (The purity of the  $t\bar{t}$  polarization can be further improved by polarizing the electron beam.) Furthermore, the top quark is boosted more in a 1 TeV machine thereby allowing a better determination of its polar angle in the  $t\bar{t}$  system because it is easier to find the right  $b$  associated with the lepton to reconstruct the top quark moving direction.

Finally, we remark that at the NLC  $\kappa_L^{\text{CC}}$  and  $\kappa_R^{\text{CC}}$  can be studied either from the decay of the top quark pair or from the single-top quark production process,  $W$ -photon fusion process  $e^-e^+(W\gamma) \rightarrow tX$ , or  $e^-\gamma(W\gamma) \rightarrow \bar{t}X$ , which is similar to the  $W$ -gluon fusion process in hadron collisions.

## 6 Discussions and Conclusions

In this lecture we have applied the electroweak chiral Lagrangian to probe new physics beyond the SM through studying the couplings of the top quark to gauge bosons. We first examined the precision LEP data to extract the information on these couplings, then we discussed how to improve our knowledge at current and future electron colliders such as at the SLC and the NLC.

Because of the non-renormalizability of the electroweak chiral Lagrangian we can only estimate the size of these nonstandard couplings by studying the contributions to LEP observables at the order of  $m_t^2 \ln \Lambda^2$ , where  $\Lambda$  ( $= 4\pi v \sim 3 \text{ TeV}$ ) is the cutoff scale of the effective Lagrangian. Already we found interesting constraints on these couplings.

Assuming  $b$ - $b$ - $Z$  vertex is not modified, we found that  $\kappa_L^{\text{NC}}$  is already constrained to be  $-0.3 < \kappa_L^{\text{NC}} < 0.6$  ( $-0.2 < \kappa_L^{\text{NC}} < 0.5$ ) by LEP data at the 95% CL. for a 150

(175) GeV top quark. Although  $\kappa_R^{\text{NC}}$  and  $\kappa_L^{\text{CC}}$  are allowed to be in the full range of  $\pm 1$ , the precision LEP data do impose some correlations among  $\kappa_L^{\text{NC}}$ ,  $\kappa_R^{\text{NC}}$ , and  $\kappa_L^{\text{CC}}$ . ( $\kappa_R^{\text{CC}}$  does not contribute to the LEP observables of interest in the limit of  $m_b = 0$ .) In our calculations, these nonstandard couplings are only inserted once in loop diagrams using dimensional regularization.

Inspired by the experimental fact  $\rho \approx 1$ , reflecting the existence of an approximate custodial symmetry, we proposed an effective model to relate  $\kappa_L^{\text{NC}}$  and  $\kappa_L^{\text{CC}}$ . We found that the nonuniversal interactions of the top quark to gauge bosons parameterized by  $\kappa_L^{\text{NC}}$ ,  $\kappa_R^{\text{NC}}$ , and  $\kappa_L^{\text{CC}}$  are well constrained by LEP data, within 95% CL. Also, the two parameters  $\kappa_L = \kappa_L^{\text{NC}}$  and  $\kappa_R = \kappa_R^{\text{NC}}$  are strongly correlated. (In our model,  $\kappa_L \sim 2\kappa_R$ .)

We note that the relations among  $\kappa$ 's can be used to test different models of electroweak symmetry-breaking. For instance, a heavy SM Higgs boson ( $m_H > m_t$ ) will modify the couplings  $t$ - $t$ - $Z$  and  $t$ - $b$ - $W$  of a heavy top quark at the scale  $m_t$  such that  $\kappa_L^{\text{NC}} = 2\kappa_L^{\text{CC}}$ ,  $\kappa_L^{\text{NC}} = -\kappa_R^{\text{NC}}$ , and  $\kappa_R^{\text{CC}} = 0$ .

It is also interesting to note that the upper bound on the top quark mass can be raised from the SM bound  $m_t < 200$  GeV to as large as 300 GeV if new physics occurs. That is to say, if there is new physics associated with the top quark, it is possible that the top quark is heavier than what the SM predicts, a similar conclusion was reached in Ref. [10].

With a better measurement of  $A_{LR}$  at the SLC, more constraint can be set on the correlation between  $\kappa_L$  and  $\kappa_R$ . To constrain the size of  $\kappa_L$  and  $\kappa_R$ , we need a more precise measurement on the partial decay width  $\Gamma(Z \rightarrow b\bar{b})$ .

Undoubtedly, direct detection of the top quark at the NLC is crucial to measuring the couplings of  $t$ - $b$ - $W$  and  $t$ - $t$ - $Z$ . The NLC shall be the best machine to measure  $\kappa_L^{\text{NC}}$  and  $\kappa_R^{\text{NC}}$  from studying the angular distribution and the polarization of the top quark produced in  $e^-e^+$  collision [37].

# LECTURE TWO:

## Heavy Top Quark Effects To Low Energy Data In The EW Chiral Lagrangian

### 1 Introduction

In Lecture One we studied the general (non-standard) couplings of the top quark to the electroweak (EW) gauge bosons in an effective chiral Lagrangian formulated electroweak theory with the spontaneously broken symmetry  $SU(2)_L \times U(1)_Y / U(1)_{em}$ . We found that from the previously announced LEP data [1] there were still considerable rooms allowed to accommodate such non-standard interactions [34]. The question regarding the origin of such non-standard interactions is of a great importance, and was discussed to some extent in the previous lecture.

In this lecture we will concentrate on two main points. The first point is to take a different approach from that used in the previous lecture to study the leading contributions of a heavy top quark (in powers of  $m_t$ ) to low energy observables. In Lecture One we calculated the leading corrections of  $\mathcal{O}(m_t^2 \ln \Lambda^2)$  arising from some non-standard couplings of the top quark to the EW gauge bosons, parameterized in the chiral Lagrangian.<sup>6</sup> The set of Feynman diagrams calculated in Lecture One contained external gauge boson lines and its internal lines could be gauge bosons and/or Goldstone bosons, in addition to fermions, to form a gauge invariant set. Because the leading corrections (in powers of  $m_t$ ) are closely related to the spontaneously symmetry-breaking (SSB) sector, we expect such corrections to be obtained from the interactions of the top quark to the Goldstone bosons alone. We shall develop a formalism to calculate these leading corrections from the pure scalar sector in the chiral Lagrangian. We show how to reproduce the results obtained in Lecture One from a set of Feynman diagrams which only contain scalar boson and fermion lines, gauge boson lines are however not needed. The relation between these two sets of Green's functions, one set is involving only the gauge bosons as external lines and the other set with external scalar boson lines in the limit of turning off the weak gauge coupling  $g$ , was derived in Ref. [38] for the SM. As to be shown in sec. 3, the relation between these two sets of Green's functions in the chiral Lagrangian can be easily derived, and its derivation is far more clear than that in the SM in which the  $SU(2)_L \times U(1)_Y$  symmetry is linearly realized. This is another example indicating the power and elegance of the non-linearly realized chiral Lagrangian approach.

The second point is to update the constraints on the non-standard couplings of the top quark to the EW gauge bosons using the new LEP [3] and SLC data [4]. The rest of this lecture is organized as follows. Sec. 2 is devoted to study the large top quark mass contribution (in powers of  $m_t$ ) to low energy physics through the

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<sup>6</sup>  $\Lambda$  is the cutoff scale at which the effective Lagrangian is valid.



quantities  $\rho$  and  $\tau$  [38] in the chiral Lagrangian formulation. In sec. 3 we update the constraints on the non-standard couplings of the top quark to the EW gauge bosons. Our previous constraints were given in Lecture One. Sec. 4 contains some conclusions.

## 2 Large Top Quark Mass Effects To Low Energy Physics

In the Sec. 2 of Lecture One we have extensively discussed how to construct a chiral Lagrangian formulated electroweak theory in which the gauge symmetry  $SU(2)_L \times U(1)_Y$  is non-linearly realized. In this lecture, we will explore an equivalent but different formulation.

Define

$$\mathcal{W}_\mu^a = -i\text{Tr}(\tau^a \Sigma^\dagger D_\mu \Sigma) \quad (77)$$

and

$$\mathcal{B}_\mu = g' B_\mu , \quad (78)$$

where

$$D_\mu \Sigma = \left( \partial_\mu - ig \frac{\tau^a}{2} W_\mu^a \right) \Sigma . \quad (79)$$

$W_\mu^a$  and  $B_\mu$  are the gauge bosons associated with the  $SU(2)_L$  and  $U(1)_Y$  groups, respectively.  $g$  and  $g'$  are the corresponding gauge couplings. These fields transform under G as

$$\mathcal{W}_\mu^3 \rightarrow \mathcal{W}_\mu'^3 = \mathcal{W}_\mu^3 - \partial_\mu y , \quad (80)$$

$$\mathcal{W}_\mu^\pm \rightarrow \mathcal{W}_\mu'^\pm = e^{\pm iy} \mathcal{W}_\mu^\pm , \quad (81)$$

$$\mathcal{B}_\mu \rightarrow \mathcal{B}_\mu' = \mathcal{B}_\mu + \partial_\mu y , \quad (82)$$

where

$$\mathcal{W}_\mu^\pm = \frac{\mathcal{W}_\mu^1 \mp i\mathcal{W}_\mu^2}{\sqrt{2}} . \quad (83)$$

Introduce the fields  $\mathcal{Z}_\mu$  and  $\mathcal{A}_\mu$  as

$$\mathcal{Z}_\mu = \mathcal{W}_\mu^3 + \mathcal{B}_\mu , \quad (84)$$

$$s^2 \mathcal{A}_\mu = s^2 \mathcal{W}_\mu^3 - c^2 \mathcal{B}_\mu , \quad (85)$$

where  $s^2 \equiv \sin^2 \theta_W$ , and  $c^2 = 1 - s^2$ . In the unitary gauge ( $\Sigma = 1$ )

$$\mathcal{W}_\mu^a = -g W_\mu^a , \quad (86)$$

$$\mathcal{Z}_\mu = -\frac{g}{c} Z_\mu , \quad (87)$$

$$\mathcal{A}_\mu = -\frac{e}{s^2} A_\mu , \quad (88)$$

where we have used the relations  $e = gs = g'c$ ,  $W_\mu^3 = cZ_\mu + sA_\mu$ , and  $B_\mu = -sZ_\mu + cA_\mu$ . The transformations of  $\mathcal{Z}_\mu$  and  $\mathcal{A}_\mu$  under  $G$  are

$$\mathcal{Z}_\mu \rightarrow \mathcal{Z}'_\mu = \mathcal{Z}_\mu , \quad (89)$$

$$\mathcal{A}_\mu \rightarrow \mathcal{A}'_\mu = \mathcal{A}_\mu - \frac{1}{g^2} \partial_\mu y . \quad (90)$$

Hence, under  $G$ , the fields  $\mathcal{W}_\mu^\pm$  and  $\mathcal{Z}_\mu$  transform as vector fields, but  $\mathcal{A}_\mu$  transforms as a gauge boson field which plays the role of the photon field  $A_\mu$ .

Out of the fields defined as above, one may construct the  $SU(2)_L \times U(1)_Y$  gauge invariant interaction terms in the chiral Lagrangian

$$\begin{aligned} \mathcal{L}^B &= -\frac{1}{4g^2} \mathcal{W}_{\mu\nu}^a \mathcal{W}^{a\mu\nu} - \frac{1}{4g'^2} \mathcal{B}_{\mu\nu} \mathcal{B}^{\mu\nu} \\ &+ \frac{v^2}{4} \mathcal{W}_\mu^+ \mathcal{W}^{-\mu} + \frac{v^2}{8} \mathcal{Z}_\mu \mathcal{Z}^\mu + \dots , \end{aligned} \quad (91)$$

where

$$\mathcal{W}_{\mu\nu}^a = \partial_\mu \mathcal{W}_\nu^a - \partial_\nu \mathcal{W}_\mu^a + \epsilon^{abc} \mathcal{W}_\mu^b \mathcal{W}_\nu^c , \quad (92)$$

$$\mathcal{B}_{\mu\nu} = \partial_\mu \mathcal{B}_\nu - \partial_\nu \mathcal{B}_\mu , \quad (93)$$

and where  $\dots$  denotes other possible four- or higher- dimensional operators [31, 25].

It is easy to show that<sup>7</sup>

$$\mathcal{W}_{\mu\nu}^a \tau^a = -g \Sigma^\dagger W_{\mu\nu}^a \tau^a \Sigma \quad (94)$$

and

$$\mathcal{W}_{\mu\nu}^a \mathcal{W}^{a\mu\nu} = g^2 W_{\mu\nu}^a W^{a\mu\nu} . \quad (95)$$

This simply reflects the fact that this kinetic term is not related to the Goldstone bosons sector, *i.e.*, it does not originate from the symmetry-breaking sector. In other words, if one is interested in the full loop corrections which include corrections of the order  $g$ , then we cannot relate these corrections (in powers of  $g$ ) entirely to the pure scalar sector.

The mass terms in Eq. (91) can be expanded as

$$\begin{aligned} \frac{v^2}{4} \mathcal{W}_\mu^+ \mathcal{W}^{-\mu} + \frac{v^2}{8} \mathcal{Z}_\mu \mathcal{Z}^\mu &= \partial_\mu \phi^+ \partial^\mu \phi^- + \frac{1}{2} \partial_\mu \phi^3 \partial^\mu \phi^3 \\ &+ \frac{g^2 v^2}{4} W_\mu^+ W^{\mu-} + \frac{g^2 v^2}{8c^2} Z_\mu Z^\mu + \dots . \end{aligned} \quad (96)$$

At the tree level, the mass of  $W^\pm$  boson is  $M_W = gv/2$  and the mass of  $Z$  boson is  $M_Z = gv/2c$ .

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<sup>7</sup> Use  $\mathcal{W}_\mu^a \tau^a = -2i \Sigma^\dagger D_\mu \Sigma$ , and  $[\tau^a, \tau^b] = 2i \epsilon^{abc} \tau^c$ .

Fermions can be included in this context by assuming that each flavor transforms under  $G = SU(2)_L \times U(1)_Y$  as [27]

$$f \rightarrow f' = e^{iyQ_f} f, \quad (97)$$

where  $Q_f$  is the electromagnetic charge of  $f$ .

Out of the fermion fields  $f_1, f_2$  (two different flavors), and the Goldstone bosons matrix field  $\Sigma$ , the usual linearly realized fields  $\Psi$  can be constructed. For example, the left-handed fermions ( $SU(2)_L$  doublet) are

$$\Psi_L = \Sigma F_L = \Sigma \begin{pmatrix} f_1 \\ f_2 \end{pmatrix}_L \quad (98)$$

with  $Q_{f_1} - Q_{f_2} = 1$ . One can easily show that  $\Psi_L$  transforms under  $G$  linearly as

$$\Psi_L \rightarrow \Psi'_L = g \Psi_L, \quad (99)$$

where  $g = \exp(i\frac{\alpha^a \tau^a}{2}) \exp(i\frac{y}{2}) \in G$ . Linearly realized right-handed fermions  $\Psi_R$  ( $SU(2)_L$  singlet) simply coincide with  $F_R$ , *i.e.*,

$$\Psi_R = F_R = \begin{pmatrix} f_1 \\ f_2 \end{pmatrix}_R. \quad (100)$$

It is then straightforward to construct a chiral Lagrangian containing both the bosonic and the fermionic fields defined as above.

Our goal is to study the large Yukawa corrections to the low energy data from the chiral Lagrangian formulated electroweak theories. We shall separate the radiative corrections as an expansion in both the Yukawa coupling  $g_t$  and the weak coupling  $g$ . ( $g_t = \sqrt{2}m_t/v$ , where  $m_t$  is the mass of the top quark.) With this separation we can then consider the case of ignoring the corrections of the order  $g$  as compared to that of  $g_t$ . This kind of study has been done in Ref. [38] for the SM, where one can concentrate on the pure scalar sector and treat the gauge bosons as classical fields so that the full gauge invariance of the SM Lagrangian is maintained and a set of Ward identities can be derived to relate the Green's functions of the Goldstone boson and the gauge boson sectors. Hence, large  $g_t$  corrections can be easily obtained from calculating Feynman diagrams involving only fermions and scalar bosons (*e.g.*, Goldstone bosons and Higgs boson) but not gauge bosons. In the chiral Lagrangian such approach is far more obvious and clear.

Why is the chiral Lagrangian formulation useful in finding large  $g_t$  corrections beyond tree level? After constructing the gauge invariant bare Lagrangian, we can perform the necessary loop calculations to any order and organize all the loop corrections in a compact form, which possesses a simple  $U(1)_{em}$  invariance, using the composite fields  $\mathcal{W}_\mu^\pm, \mathcal{Z}_\mu$ , and  $\mathcal{A}_\mu$ . In principle one needs to fix a gauge to perform loop calculations. Fixing a gauge will however destroy the gauge invariance of the Lagrangian. This is true because the gauge fixing term (*e.g.*, in  $R_\xi$  gauge) will explicitly break gauge invariance. Since we are interested in large  $g_t$  corrections, we do

not need to consider gauge bosons in loops [38]. This means that we do not need to fix a gauge and thus we can maintain the full gauge invariance of the effective Lagrangian. This is obvious by observing that the leading contributions enhanced by powers of  $m_t$  are clearly products of the SSB and have nothing to do with the weak gauge coupling  $g$ . The point is that because the final result can be organized in the gauge invariant way as described above we can immediately notice the equivalence between the two sets of calculations, *i.e.*, using the Goldstone bosons and using the gauge bosons. From the expansion of the field

$$\mathcal{Z}_\mu = \frac{2}{v} \partial_\mu \phi^3 - \frac{g}{c} Z_\mu + \dots, \quad (101)$$

one notices that each gauge boson field has a factor  $g$  in front. Hence, if we are interested in corrections independent of the gauge coupling  $g$ , we need only to consider the pure scalar sector.

## 2.1 Effective Lagrangian

To obtain the large contributions of the top quark mass (in powers of  $m_t$ ) to low energy data, we need only to concentrate on the top-bottom fermionic sector (with  $f_1 = t$  and  $f_2 = b$ ) in addition to the bosonic sector. The most general gauge invariant chiral Lagrangian can be written as

$$\begin{aligned} \mathcal{L}_0 = & i\bar{t}\gamma^\mu \left( \partial_\mu + i\frac{2s_0^2}{3}\mathcal{A}_\mu \right) t + i\bar{b}\gamma^\mu \left( \partial_\mu - i\frac{s_0^2}{3}\mathcal{A}_\mu \right) b \\ & - \left( \frac{1}{2} - \frac{2s_0^2}{3} + \kappa_L^{\text{NC}} \right) \bar{t}_L\gamma^\mu t_L \mathcal{Z}_\mu - \left( \frac{-2s_0^2}{3} + \kappa_R^{\text{NC}} \right) \bar{t}_R\gamma^\mu t_R \mathcal{Z}_\mu \\ & - \left( \frac{-1}{2} + \frac{s_0^2}{3} \right) \bar{b}_L\gamma^\mu b_L \mathcal{Z}_\mu - \frac{s_0^2}{3} \bar{b}_R\gamma^\mu b_R \mathcal{Z}_\mu \\ & - \frac{1}{\sqrt{2}} (1 + \kappa_L^{\text{CC}}) \bar{t}_L\gamma^\mu b_L \mathcal{W}_\mu^+ - \frac{1}{\sqrt{2}} (1 + \kappa_L^{\text{CC}\dagger}) \bar{b}_L\gamma^\mu t_L \mathcal{W}_\mu^- \\ & - \frac{1}{\sqrt{2}} \kappa_R^{\text{CC}} \bar{t}_R\gamma^\mu b_R \mathcal{W}_\mu^+ - \frac{1}{\sqrt{2}} \kappa_R^{\text{CC}\dagger} \bar{b}_R\gamma^\mu t_R \mathcal{W}_\mu^- \\ & - m_t \bar{t}t + \dots, \end{aligned} \quad (102)$$

where  $\kappa_L^{\text{NC}}$ ,  $\kappa_R^{\text{NC}}$ ,  $\kappa_L^{\text{CC}}$ , and  $\kappa_R^{\text{CC}}$  parameterize possible deviations from the SM predictions [34], and  $\dots$  indicates possible Higgs boson interactions and all possible higher dimensional operators. Here we have assumed that new physics from the SSB sector might modify the interactions of the top quark to gauge bosons due to the heavy mass of the top quark, but the bare  $b$ - $b$ - $Z$  couplings are not modified in the limit of ignoring the mass of the bottom quark [34]. The subscript 0 denotes bare quantities and all the fields in the Lagrangian  $\mathcal{L}_0$ , Eq. (19), are bare fields.

Needless to say, the composite fields are only used to organize the radiative corrections in the chiral Lagrangian. To actually calculate loop corrections one should

expand these operators in terms of the Goldstone boson and the gauge boson fields. The gauge invariant result of loop calculations can be written in a form similar to Eq. (19). Denoting this effective Lagrangian as  $\mathcal{L}_{eff}$ , then its fermionic sector takes the following form:

$$\begin{aligned}\mathcal{L}_{eff} = & iZ_b^L \overline{b_L} \gamma_\mu \partial^\mu b_L + Z_1 \frac{s_0^2}{3} \overline{b_L} \gamma_\mu b_L \mathcal{A}^\mu + \frac{1}{2} \left( Z_v^L - Z_2 \frac{2s_0^2}{3} \right) \overline{b_L} \gamma_\mu b_L \mathcal{Z}^\mu \\ & + iZ_b^R \overline{b_R} \gamma_\mu \partial^\mu b_R + Z_3 \frac{s_0^2}{3} \overline{b_R} \gamma_\mu b_R \mathcal{A}^\mu - Z_4 \frac{s_0^2}{3} \overline{b_R} \gamma_\mu b_R \mathcal{Z}^\mu + \dots, \quad (103)\end{aligned}$$

in which the coefficient functions  $Z_1$ ,  $Z_2$ ,  $Z_3$ , and  $Z_4$  contain all the loop results and as in  $\mathcal{L}_0$  all the fields in  $\mathcal{L}_{eff}$  are bare fields.

In the case of ignoring the corrections of the order  $g$  the effective Lagrangian can be further separated into two parts: one part has the explicit linear  $U(1)_Y$  symmetry in the unitary gauge, another contains all the radiative corrections which do not vanish when taking the  $g \rightarrow 0$  limit. Namely, in this approximation, we can write

$$\begin{aligned}\mathcal{L}_{eff} = & iZ_b^L \overline{b_L} \gamma_\mu \partial^\mu b_L - Z_1 \frac{1}{3} \overline{b_L} \gamma_\mu b_L \mathcal{B}^\mu + \frac{1}{2} Z_v^L \overline{b_L} \gamma_\mu b_L \mathcal{Z}^\mu \\ & + iZ_b^R \overline{b_R} \gamma_\mu \partial^\mu b_R - Z_3 \frac{1}{3} \overline{b_R} \gamma_\mu b_R \mathcal{B}^\mu + \dots, \quad (104)\end{aligned}$$

where

$$\mathcal{B}_\mu = s_0^2 (\mathcal{Z}_\mu - \mathcal{A}_\mu), \quad (105)$$

derived from Eqs. (84) and (85). Note that as shown in Eqs. (78) and (82) the field  $\mathcal{B}_\mu$  is not composite and transforms exactly like  $B_\mu$ . Comparing Eq. (103) with (104), we conclude that the coefficient functions  $Z_1$ ,  $Z_2$ ,  $Z_3$ , and  $Z_4$  must be related and

$$Z_2 = Z_1, \quad (106)$$

$$Z_4 = Z_3. \quad (107)$$

All the radiative corrections to the vertex  $b\text{-}b\text{-}\phi^3$  in powers of  $m_t$  are summarized by the coefficient function  $Z_v^L$  because, from Eq. (101),

$$\frac{1}{2} Z_v^L \overline{b_L} \gamma_\mu b_L \mathcal{Z}^\mu = Z_v^L \frac{1}{v} \overline{b_L} \gamma_\mu b_L \partial^\mu \phi^3 + \dots. \quad (108)$$

Since the effective Lagrangian  $\mathcal{L}_{eff}$  must possess an explicit  $U(1)_{em}$  symmetry and under G the field  $\mathcal{A}_\mu$  transforms as a gauge boson field while the field  $\mathcal{Z}_\mu$  transforms as a neutral vector boson field, therefore, based upon the Ward identities in QED we conclude that in Eq. (103)

$$Z_1 = Z_b^L, \quad (109)$$

and

$$Z_3 = Z_b^R. \quad (110)$$

Hence, the effective Lagrangian  $\mathcal{L}_{eff}$  can be rewritten as

$$\begin{aligned}\mathcal{L}_{eff} &= iZ_b^L \overline{b_L} \gamma^\mu \left( \partial_\mu - i \frac{s_0^2}{3} \mathcal{A}_\mu \right) b_L + iZ_b^R \overline{b_R} \gamma^\mu \left( \partial_\mu - i \frac{s_0^2}{3} \mathcal{A}_\mu \right) b_R \\ &+ \frac{1}{2} \left( \frac{Z_v^L}{Z_b^L} - \frac{2s_0^2}{3} \right) \overline{b_L} \gamma_\mu b_L \mathcal{Z}^\mu - Z_b^R \frac{s_0^2}{3} \overline{b_R} \gamma_\mu b_R \mathcal{Z}^\mu + \dots\end{aligned}\quad (111)$$

This effective Lagrangian summarizes all the loop corrections in powers of  $m_t$  in the coefficient functions  $Z_b^L$ ,  $Z_b^R$ , and  $Z_v^L$ . Recall that up to here all the fields in  $\mathcal{L}_{eff}$  are bare fields. To make use of the effective Lagrangian to extract out the information on low energy data we prefer to express  $\mathcal{L}_{eff}$  in terms of the renormalized fields. After inspecting Eq. (111) it is clear that the kinetic terms associated with the  $b_L$  and  $b_R$  fields can be properly normalized to make the residue of their propagators to be unity by redefining (renormalizing) the fields  $b_L$  and  $b_R$  by  $(Z_b^L)^{\frac{1}{2}} b_L$  and  $(Z_b^R)^{\frac{1}{2}} b_R$ , respectively. In terms of the renormalized fields  $b_L$  and  $b_R$ ,  $\mathcal{L}_{eff}$  can be rewritten as

$$\begin{aligned}\mathcal{L}_{eff} &= \overline{b_L} i \gamma^\mu \left( \partial_\mu - i \frac{s_0^2}{3} \mathcal{A}_\mu \right) b_L + \overline{b_R} i \gamma^\mu \left( \partial_\mu - i \frac{s_0^2}{3} \mathcal{A}_\mu \right) b_R \\ &+ \frac{1}{2} \left( \frac{Z_v^L}{Z_b^L} - \frac{2s_0^2}{3} \right) \overline{b_L} \gamma_\mu b_L \mathcal{Z}^\mu - \frac{s_0^2}{3} \overline{b_R} \gamma_\mu b_R \mathcal{Z}^\mu + \dots\end{aligned}\quad (112)$$

Before considering the physical observables to low energy data let us first examine the bosonic sector. Similar to our previous discussions, loop corrections to the bosonic sector can be organized by the effective Lagrangian

$$\begin{aligned}\mathcal{L}_{eff}^B &= -\frac{1}{4g_0^2} \mathcal{W}_{\mu\nu}^a \mathcal{W}^{\mu\nu a} - \frac{1}{4g_0'^2} \mathcal{B}_{\mu\nu} \mathcal{B}^{\mu\nu} \\ &+ Z^\phi \frac{v_0^2}{4} \mathcal{W}_\mu^+ \mathcal{W}^{-\mu} + Z^\chi \frac{v_0^2}{8} \mathcal{Z}_\mu \mathcal{Z}^\mu + \dots\end{aligned}\quad (113)$$

We note that in the above equation we have explicitly used the subscript 0 to indicate bare quantities. The bosonic Lagrangian in Eq. (91) and the identity in Eq. (95) imply that the Yang-Mills terms (the first two terms in  $\mathcal{L}^B$ ) are not directly related to the SSB sector. Hence, any radiative corrections to the field  $W_{\mu\nu}^a$  must know about the weak coupling  $g$ , *i.e.*, suppressed by  $g$  in our point of view. This also holds for operators, of dimension four or higher, including  $W_{\mu\nu}^a$  in the chiral Lagrangian formulated electroweak theories where these gauge invariant terms are all suppressed by the weak coupling  $g$  [31, 25]. The same conclusion applies to  $B_{\mu\nu}$ . Therefore we conclude that the fields  $\mathcal{W}_\mu^\pm$ ,  $\mathcal{Z}_\mu$ , and  $\mathcal{A}_\mu$  in  $\mathcal{L}_{eff}$  and  $\mathcal{L}_{eff}^B$  do not get wavefunction corrections (renormalization) in the limit of ignoring all the corrections of the order  $g$ , namely the renormalized fields and the bare fields are identical in this limit.

Expanding the mass terms in Eq. (113) we find

$$\begin{aligned}Z^\phi \frac{v_0^2}{4} \mathcal{W}_\mu^+ \mathcal{W}^{-\mu} + Z^\chi \frac{v_0^2}{8} \mathcal{Z}_\mu \mathcal{Z}^\mu &= Z^\phi \partial_\mu \phi^+ \partial^\mu \phi^- + \frac{1}{2} Z^\chi \partial_\mu \phi^3 \partial^\mu \phi^3 \\ &+ Z^\phi \frac{g_0^2 v_0^2}{4} W_\mu^+ W^{-\mu} + Z^\chi \frac{g_0^2 v_0^2}{8c_0^2} Z_\mu Z^\mu + \dots\end{aligned}\quad (114)$$

It becomes clear that  $Z^\phi$  denotes the self energy corrections of the charged Goldstone boson  $\phi^\pm$ , and  $Z^\chi$  denotes the self energy corrections of the neutral Goldstone boson  $\phi^3$ . Since  $W_\mu^\pm$  and  $Z_\mu$  do not get wavefunction correction in powers of  $m_t$ , therefore the gauge boson masses are

$$\begin{aligned} M_W^2 &= Z^\phi \frac{g_0^2 v_0^2}{4} = Z^\phi M_{W_0}^2 , \\ M_Z^2 &= Z^\chi \frac{g_0^2 v_0^2}{4c_0^2} = Z^\chi M_{Z_0}^2 . \end{aligned} \quad (115)$$

In summary, all the loop corrections in powers of  $m_t$  to low energy data can be organized in the sum of  $\mathcal{L}_{eff}$  (in Eq. (112)) and  $\mathcal{L}_{eff}^B$  (in Eq. (113)). Comparing them to the bare Lagrangian  $\mathcal{L}_0$  in Eq. (19), we found that in the limit of taking  $g \rightarrow 0$  the chiral Lagrangian  $\mathcal{L}_0$  behaves as a renormalizable theory although in general a chiral Lagrangian is non-renormalizable. In other words, no higher dimensional operators (counterterms) are needed to renormalize the theory in this limit. The same feature was also found in another application of a chiral Lagrangian with  $1/N$  expansion [39].

## 2.2 Renormalization

Now we are ready to consider the large  $m_t$  contributions to low energy data. We choose our renormalization scheme to be the  $\alpha$ ,  $G_F$ , and  $M_Z$  scheme. With

$$g_0^2 = \frac{4\pi\alpha_0}{s_0^2} \quad (116)$$

and

$$s_0^2 c_0^2 = \frac{\pi\alpha_0}{\sqrt{2}G_{F0}M_{Z0}^2} , \quad (117)$$

or,

$$s_0^2 = \frac{1}{2} \left[ 1 - \left( 1 - \frac{4\pi\alpha_0}{\sqrt{2}G_{F0}M_{Z0}^2} \right)^{1/2} \right] . \quad (118)$$

Define the counterterms as

$$\begin{aligned} \alpha &= \alpha_0 + \delta\alpha , \\ G_F &= G_{F0} + \delta G_F , \\ M_Z^2 &= M_{Z0}^2 + \delta M_Z^2 , \end{aligned} \quad (119)$$

and

$$\begin{aligned} s^2 &= s_0^2 + \delta s^2 = s_0^2 - \delta c^2 , \\ c^2 &= c_0^2 + \delta c^2 , \end{aligned} \quad (120)$$

then

$$s^2 c^2 + (c^2 - s^2) \delta c^2 = \frac{\pi\alpha}{\sqrt{2}G_F M_Z^2} \left( 1 - \frac{\delta\alpha}{\alpha} + \frac{\delta G_F}{G_F} + \frac{\delta M_Z^2}{M_Z^2} \right) . \quad (121)$$

As shown in the above equation, even after the counterterms  $\delta\alpha$ ,  $\delta G_F$ , and  $\delta M_Z^2$  are fixed by data (*e.g.*, the electron (g-2), muon lifetime, and the mass of the  $Z$  boson), we still have the freedom to choose what  $\delta c^2$  is by defining differently the renormalized quantity  $s^2 c^2$ . In our case we would choose the definition of the renormalized  $s^2$  such that there will be no large top quark mass dependence (in powers of  $m_t$ ) in the counterterm  $\delta c^2$ . We shall show later that for this purpose our renormalized  $s^2$  will satisfy<sup>8</sup>

$$s^2 c^2 = \frac{\pi\alpha}{\sqrt{2}G_F M_Z^2 \rho} , \quad (122)$$

where  $\rho$  is defined from the partial width of  $Z$  into lepton pair, cf. Eq. (130). With this choice of  $s^2$  and the definition of the renormalized weak coupling

$$g^2 = \frac{4\pi\alpha}{s^2} , \quad (123)$$

one can easily show that the counterterm  $\delta g^2$  ( $= g^2 - g_0^2$ ) does not contain large  $m_t$  dependence either. (Obviously,  $\delta\alpha$  will not have contributions purely in powers of  $m_t$ .) Namely, in this renormalization scheme,  $\alpha$ ,  $g$ , and  $s^2$  do not get renormalized after ignoring all the contributions of the order  $g$ . Hence, all the bare couplings  $g_0$ ,  $g'_0$ , and  $s_0^2$  in the effective Lagrangians  $\mathcal{L}_{eff}$  and  $\mathcal{L}_{eff}^B$  do not get corrected when considering the contributions which do not vanish in the limit of  $g \rightarrow 0$ . The only non-vanishing counterterm needs to be considered in Eq. (113) is  $\delta v^2$  ( $= v^2 - v_0^2$ ), which can be obtained by noting that neither  $g$  nor  $\mathcal{W}^\pm$  (or  $W^\pm$ ) gets renormalized. Therefore, from Eq. (115),

$$Z^\phi v_0^2 = v^2 . \quad (124)$$

Thus

$$G_{F0} = \frac{1}{\sqrt{2}v_0^2} = Z^\phi \frac{1}{\sqrt{2}v^2} = Z^\phi G_F . \quad (125)$$

Consequently,

$$\frac{g_0^2}{c_0^2} = \frac{8G_{F0}M_{Z0}^2}{\sqrt{2}} = \frac{8G_F M_Z^2}{\sqrt{2}} \frac{Z^\phi}{Z^\chi} \quad (126)$$

and the effective  $Z$ - $b$ - $b$  coupling is

$$-\frac{g_0}{2c_0}\gamma^\mu \left[ \left( \frac{Z_v^L}{Z_b^L} - \frac{2s_0^2}{3} \right) P_L - \frac{2s_0^2}{3} P_R \right] = -\sqrt{\frac{G_F M_Z^2}{2\sqrt{2}} \frac{Z^\phi}{Z^\chi}} \gamma^\mu \left[ \left( \frac{Z_v^L}{Z_b^L} - \frac{4s^2}{3} \right) - \frac{Z_v^L}{Z_b^L} \gamma_5 \right] , \quad (127)$$

where  $P_{L,R} = (1 \mp \gamma_5)/2$ .

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<sup>8</sup> If we define  $s'^2 c'^2 = \frac{\pi\alpha}{\sqrt{2}G_F M_Z^2}$ , then  $s^2 = s'^2(1 + \Delta k')$  with  $\Delta k' = \frac{-c'^2 \delta \rho}{c'^2 - s'^2}$ , and the counterterm of  $s'^2$  will contain contributions in powers of  $m_t$ .



## 2.3 Low Energy Observables

In general, all the radiative corrections to low energy data can be categorized in a model independent way into four parameters:  $S$ ,  $T$ ,  $U$  [13], and  $R_b$  [40]; or equivalently,  $\epsilon_1$ ,  $\epsilon_2$ ,  $\epsilon_3$ , and  $\epsilon_b$  [14].

These parameters are derived from four basic measured observables,  $\Gamma_\mu$  (the partial decay width of  $Z$  into a  $\mu$  pair),  $A_{FB}^\mu$  (the forward-backward asymmetry at the  $Z$  peak for the  $\mu$  lepton),  $M_W/M_Z$  (the ratio of  $W^\pm$  and  $Z$  masses), and  $\Gamma_b$  (the partial decay width of  $Z$  into a  $b\bar{b}$  pair). The expressions of these observables in terms of  $\epsilon$ 's are given in Ref. [14]. In this lecture we only give the relevant terms in  $\epsilon$ 's which might contain the leading effects in powers of  $m_t$  from new physics. The relations between these two sets of parameters are, to the order of interest,

$$\begin{aligned} S &= \frac{4s^2}{\alpha(M_Z^2)}\epsilon_3, \\ T &= \frac{1}{\alpha(M_Z^2)}\epsilon_1, \\ U &= \frac{-4s^2}{\alpha(M_Z^2)}\epsilon_2, \end{aligned} \tag{128}$$

and both  $R_b (= \Gamma_b/\Gamma_h)$  and  $\epsilon_b$  are measuring the effects of new physics in the partial decay width of  $Z \rightarrow b\bar{b}$ . ( $\Gamma_h$  is the hadronic width of  $Z$ .)

As shown in Lecture One, both  $\epsilon_1$  and  $\epsilon_b$  gain corrections in powers of  $m_t$ , and are sensitive to new physics coming through the top quark [34]. On the contrary,  $\epsilon_2$  and  $\epsilon_3$  do not play any significant role in our analysis because their dependence on the top mass is only logarithmic. Hence,

$$\begin{aligned} \epsilon_1 &= \delta\rho + \text{corrections of the order } g, \\ \epsilon_b &= \tau + \text{corrections of the order } g, \\ \epsilon_2 &= \text{corrections of the order } g, \\ \epsilon_3 &= \text{corrections of the order } g, \end{aligned} \tag{129}$$

where  $\delta\rho = \rho - 1$ . The parameters  $\rho$  and  $\tau$  are defined from

$$\begin{aligned} \Gamma_\mu &\equiv \Gamma(Z \rightarrow \mu^+\mu^-) = \rho \frac{G_F M_Z^3}{6\pi\sqrt{2}} (g_{\mu V}^2 + g_{\mu A}^2), \\ \Gamma_b &\equiv \Gamma(Z \rightarrow b\bar{b}) = \rho \frac{G_F M_Z^3}{2\pi\sqrt{2}} (g_{b V}^2 + g_{b A}^2), \end{aligned} \tag{130}$$

where

$$\begin{aligned} g_{\mu V} &= \frac{-1}{2} (1 - 4s^2), \quad g_{\mu A} = \frac{-1}{2}, \\ g_{b V} &= \frac{-1}{2} \left(1 - \frac{4}{3}s^2 + \tau\right), \quad g_{b A} = \frac{-1}{2} (1 + \tau). \end{aligned} \tag{131}$$

Figure 3: The relevant Feynman diagrams, which contribute to  $\rho$  and  $\tau$  to the order  $\mathcal{O}(m_t^2 \ln \Lambda^2)$ .

Hence, comparing to Eq. (127) we conclude

$$\begin{aligned}\delta\rho &= \frac{Z^\phi}{Z^\chi} - 1, \\ \tau &= \frac{Z_v^L}{Z_b^L} - 1.\end{aligned}\tag{132}$$

## 2.4 One Loop Corrections in the SM

The SM, being a linearly realized  $SU(2)_L \times U(1)_Y$  gauge theory, can be formulated as a chiral Lagrangian after non-linearly transforming the fields [34]. Applying the previous formalism, we can calculate one loop corrections of order  $m_t^2$  to  $\rho$  and  $\tau$  for the SM by taking  $\kappa_L^{\text{NC}} = \kappa_R^{\text{NC}} = \kappa_L^{\text{CC}} = \kappa_R^{\text{CC}} = 0$  in Eq. (19). These loop corrections can be summarized by the coefficient functions  $Z^\chi$ ,  $Z^\phi$ ,  $Z_b^L$ , and  $Z_v^L$  which are calculated from the Feynman diagrams shown in Figs. 3(a), 3(b), 3(c), and the sum of 3(d) and 3(e), respectively. We found

$$\begin{aligned}Z^\chi &= 1 + \frac{6m_t^2}{16\pi^2 v^2} (\Delta - \ln m_t^2), \\ Z^\phi &= 1 + \frac{6m_t^2}{16\pi^2 v^2} \left( \Delta + \frac{1}{2} - \ln m_t^2 \right), \\ Z_b^L &= 1 + \frac{3m_t^2}{16\pi^2 v^2} \left( -\Delta + \ln m_t^2 - \frac{5}{6} \right), \\ Z_v^L &= 1 + \frac{3m_t^2}{16\pi^2 v^2} \left( -\Delta + \ln m_t^2 - \frac{3}{2} \right).\end{aligned}\tag{133}$$

We note that Fig. 3(e) arises from the non-linear realization of the gauge symmetry in the chiral Lagrangian approach. Substituting the above results into Eq. (132), we obtain

$$\begin{aligned}\delta\rho &= \frac{3G_F m_t^2}{8\sqrt{2}\pi^2}, \\ \tau &= \frac{-G_F m_t^2}{4\sqrt{2}\pi^2},\end{aligned}\tag{134}$$

which are the established results [38].

### 3 Updating the Top Quark Couplings to the EW Gauge Bosons

In Lecture One we calculated the one-loop corrections (of order  $m_t^2 \ln \Lambda^2$ ) to  $\rho$  and  $\tau$  due to the non-standard couplings of the top quark to the EW gauge bosons by considering a set of Feynman diagrams with the external massive gauge bosons lines. In this lecture we show how to reproduce those results by considering a set of Feynman diagrams which include the pure Goldstone boson and fermion lines as described in section 2.

Inserting these non-standard couplings in loop diagrams and keeping only the linear terms in  $\kappa$ 's, we found, at the order of  $m_t^2 \ln \Lambda^2$ ,

$$\begin{aligned} Z^\chi &= 1 + \frac{6m_t^2}{16\pi^2 v^2} (2\kappa_L^{\text{NC}} - 2\kappa_R^{\text{NC}}) \ln \frac{\Lambda^2}{m_t^2} , \\ Z^\phi &= 1 + \frac{6m_t^2}{16\pi^2 v^2} (2\kappa_L^{\text{CC}}) \ln \frac{\Lambda^2}{m_t^2} , \\ Z_b^L &= 1 - \frac{6m_t^2}{16\pi^2 v^2} \kappa_L^{\text{CC}} \ln \frac{\Lambda^2}{m_t^2} , \\ Z_v^L &= 1 - \frac{m_t^2}{16\pi^2 v^2} (6\kappa_L^{\text{CC}} - 4\kappa_L^{\text{NC}} + \kappa_R^{\text{NC}}) \ln \frac{\Lambda^2}{m_t^2} . \end{aligned} \quad (135)$$

Thus the nonstandard contributions to  $\rho$  and  $\tau$  are

$$\begin{aligned} \delta\rho &= \frac{3G_F m_t^2}{2\sqrt{2}\pi^2} (\kappa_L^{\text{CC}} - \kappa_L^{\text{NC}} + \kappa_R^{\text{NC}}) \ln \frac{\Lambda^2}{m_t^2} , \\ \tau &= \frac{G_F m_t^2}{2\sqrt{2}\pi^2} \left( -\frac{1}{4}\kappa_R^{\text{NC}} + \kappa_L^{\text{NC}} \right) \ln \frac{\Lambda^2}{m_t^2} , \end{aligned} \quad (136)$$

which agree with our previous results obtained in Lecture One, cf. Eqs. (42) and (43).

Based upon the new LEP measurements [3], a global analysis indicates a SM top quark mass to be [5]

$$m_t = 165 \pm 12 \text{ GeV for } m_H = 300 \text{ GeV} . \quad (137)$$

If the SLC measurement is included with LEP measurements, then

$$m_t = 174 \pm 11 \text{ GeV for } m_H = 300 \text{ GeV} , \quad (138)$$

because the new SLC measurement of  $A_{LR}$  [4] implied a heavier top quark.

Using the new LEP and SLC results we shall update the constraints on the non-standard couplings of the top quark to the EW gauge bosons. This can be done by comparing the new experimental values for  $\delta\rho$  and  $\tau$  with that predicted by the SM

Figure 4: Two-dimensional projection in the plane of  $\kappa_R^{\text{NC}}$  and  $\kappa_L^{\text{CC}}$ , for  $m_t = 175$  GeV and  $m_H = 100$  GeV.

Figure 5: Two-dimensional projection in the plane of  $\kappa_L^{\text{NC}}$  and  $\kappa_R^{\text{NC}}$ , for  $m_t = 175$  GeV and  $m_H = 100$  GeV.

and the non-standard contributions combined. In the limit of ignoring the contributions of the order  $g$ , the observables  $\Gamma_\mu$ ,  $A_{FB}^\mu$ ,  $M_W/M_Z$ , and  $\Gamma_b$  can all be expressed in terms of the two quantities  $\delta\rho$  and  $\tau$ . In addition to Eq. (130), we found

$$A_{FB}^\mu = \frac{3g_{\mu V}^2 g_{\mu A}^2}{(g_{\mu V}^2 + g_{\mu A}^2)^2} \quad (139)$$

and<sup>9</sup>

$$\frac{M_W^2}{M_Z^2} = \rho c^2. \quad (140)$$

Using the minimum set of observables ( $\Gamma_\mu$ ,  $A_{FB}^\mu$ ,  $M_W/M_Z$ , and  $\Gamma_b$ ), we constrain the allowed space of  $\kappa$ 's in a model independent way without specifying the explicit dynamics for generating these non-standard effects. One can also enlarge the set of observables used in the analysis by including all the LEP measurements and the measurement of the left-right cross section asymmetry  $A_{LR}$  in  $Z$  production with a longitudinally polarized electron beam at the SLC, where

$$A_{LR} = \frac{2x}{1+x^2}, \quad (141)$$

with

$$x = \frac{g_{eV}}{g_{eA}} = 1 - 4s^2. \quad (142)$$

Following the same analyses carried out in the Lecture One, we include both the SM and the non-standard contributions to low energy data. The SM contributions to  $\delta\rho$  and  $\tau$  were given in Ref. [14] for various top quark and Higgs boson masses. Our conclusions however are not sensitive to the Higgs boson mass [34], as discussed in the previous lecture.

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<sup>9</sup> In terms of the quantity  $\Delta r_w$  defined in Ref. [14],  $\frac{M_W^2}{M_Z^2} \left(1 - \frac{M_W^2}{M_Z^2}\right) = \frac{\pi\alpha(M_Z^2)}{\sqrt{2}G_F M_Z^2 (1 - \Delta r_w)}$ . For corrections in powers of  $m_t$ ,  $s^2 \Delta r_w = -c^2 \delta\rho$ .

Figure 6: Two-dimensional projection in the plane of  $\kappa_L^{\text{NC}}$  and  $\kappa_L^{\text{CC}}$ , for  $m_t = 175$  GeV and  $m_H = 100$  GeV.

Choosing  $m_t = 175$  GeV and  $m_H = 100$  GeV, we span the parameter space defined by  $-1.0 \lesssim \kappa_L^{\text{NC}} \lesssim 1.0$ ,  $-1.0 \lesssim \kappa_R^{\text{NC}} \lesssim 1.0$ , and  $-1.0 \lesssim \kappa_L^{\text{CC}} \lesssim 1.0$ , and compare with the values<sup>10</sup>

$$\delta\rho = (3.5 \pm 1.8) \times 10^{-3} , \quad (143)$$

and

$$\tau = (0.9 \pm 4.2) \times 10^{-3} \quad (144)$$

from a global fit [5] using all the new LEP and SLC data. For reference, we listed in the following some of the relevant data, taken from [5],

$$\begin{aligned} \alpha^{-1}(M_Z^2) &= 128.87 \pm 0.12 , \\ G_F &= 1.16637(2) \times 10^{-5} \text{ GeV}^{-2} , \\ M_Z &= 91.1899 \pm 0.0044 \text{ GeV} , \\ M_W/M_Z &= 0.8814 \pm 0.0021 , \\ \Gamma_\ell &= 83.98 \pm 0.18 \text{ MeV} , \\ \Gamma_b &= 385.9 \pm 3.4 \text{ MeV} , \\ A_{FB}^\ell &= 0.0170 \pm 0.0016 , \\ A_{FB}^b &= 0.0970 \pm 0.0045 , \\ A_{LR}(\text{SLC}) &= 0.1668 \pm 0.079 . \end{aligned}$$

We found that within  $2\sigma$  the allowed region of these three parameters exhibits the same features as that obtained using the old set of data (see Lecture One). These features can be deduced from the two-dimensional projections of the allowed parameter space shown in Figs. 4, 5, and 6.

- (1) As a function of the top quark mass, the allowed parameter space shrinks as the top quark mass increases.
- (2) Data do not exclude possible new physics coming through the top quark coupling to the EW gauge bosons. As shown in Fig. 4,  $\kappa_L^{\text{CC}}$  and  $\kappa_R^{\text{NC}}$  are not yet constrained by the current data. Furthermore, no conclusion can be drawn regarding  $\kappa_R^{\text{CC}}$  since  $\delta\rho$  and  $\tau$  are independent of  $\kappa_R^{\text{CC}}$  at the order of  $m_t^2$ . Also we notice from
- (3)  $\kappa_L^{\text{NC}}$  is almost constrained. New physics prefers positive  $\kappa_L^{\text{NC}}$ , see Figs. 5 and 6. For example,  $\kappa_L^{\text{NC}}$  is constrained within  $(-0.3 \text{ to } 0.5)$  for a 175 GeV top quark.
- (4) New physics prefers  $\kappa_L^{\text{CC}} \approx -\kappa_R^{\text{NC}}$ . This is clearly shown in Fig. 4.

As compared with the old set of data from LEP and SLC, new data tighten the allowed region of the non-standard parameters  $\kappa$ 's by no more than a factor of two. This difference is due to the slightly smaller errors on the new measurements

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<sup>10</sup>  $\epsilon_1 = \delta\rho$ ,  $\epsilon_b = \tau$ ,  $\epsilon_2 = (-9.2 \pm 5.1) \times 10^{-3}$ , and  $\epsilon_3 = (3.8 \pm 1.9) \times 10^{-3}$ .

Figure 7: The allowed region of  $\kappa_L^{\text{NC}}$  and  $\kappa_R^{\text{NC}}$  ( $\kappa_L^{\text{NC}} = 2\kappa_L^{\text{CC}}$ ), for  $m_t = 150$  GeV and  $m_H = 100$  GeV.

Figure 8: The allowed region of  $\kappa_L^{\text{NC}}$  and  $\kappa_R^{\text{NC}}$  ( $\kappa_L^{\text{NC}} = 2\kappa_L^{\text{CC}}$ ), for  $m_t = 175$  GeV and  $m_H = 100$  GeV.

as compared with the old ones. The largest impact of these new data on our results comes from the more precise measurement of  $\Gamma_b$  which turns out to be about  $2\sigma$  higher than the SM prediction and implies a lighter top quark. For a much heavier top quark, new physics must come in because all the  $\kappa$ 's cannot simultaneously vanish. If the large discrepancy between LEP and SLC data persists, then our model of having non-standard top quark couplings to the gauge bosons is one of the candidates that can accommodate such a difference.

If we restrict ourselves to the minimum set of observables, which give [5]

$$\delta\rho = (4.8 \pm 2.2) \times 10^{-3} , \quad (145)$$

$$\tau = (5.0 \pm 4.8) \times 10^{-3} , \quad (146)$$

we reach almost the same conclusion. The main difference is that  $\kappa_L^{\text{NC}}$  shifts slightly to the right, due to the fact that the central value of  $\tau$  in this case is larger than its global fit value.

In Lecture One we discussed an effective model incorporated with an additional approximate global custodial symmetry (responsible for  $\rho = 1$  at the tree level), and concluded that  $\kappa_L^{\text{NC}} = 2\kappa_L^{\text{CC}}$  as long as the tree-level vertex  $b\text{-}b\text{-}Z$  is not modified. From Eq. (136), we found for this model

$$\delta\rho = \frac{3G_F m_t^2}{2\sqrt{2}\pi^2} \left( -\frac{1}{2}\kappa_L^{\text{NC}} + \kappa_R^{\text{NC}} \right) \ln \frac{\Lambda^2}{m_t^2} \quad (147)$$

and

$$\tau = \frac{G_F m_t^2}{2\sqrt{2}\pi^2} \left( -\frac{1}{4}\kappa_R^{\text{NC}} + \kappa_L^{\text{NC}} \right) \ln \frac{\Lambda^2}{m_t^2} . \quad (148)$$

Using this effective model, we span the plane defined by  $\kappa_L^{\text{NC}}$  and  $\kappa_R^{\text{NC}}$  for top quark mass 150 and 175 GeV, respectively. Figs. 7 and 8 show the allowed range for those parameters within  $2\sigma$ . As a general feature one observes that the allowed range forms a narrow area aligned close to the line  $\kappa_L^{\text{NC}} = 2\kappa_R^{\text{NC}}$ . For  $m_t = 150$  GeV (175 GeV) we see that  $-0.05 \lesssim \kappa_L^{\text{NC}} \lesssim 0.3$  ( $0.0 \lesssim \kappa_L^{\text{NC}} \lesssim 0.25$ ). As the top quark mass increases this range shrinks and moves downward to the right, away from the

Figure 9: The allowed range of  $\kappa_L^{\text{CC}}$  as a function of the mass of the top quark. (Note that  $\kappa_L^{\text{NC}} = 2\kappa_L^{\text{CC}}$ .)

Figure 10: The allowed range of  $\kappa_L^{\text{NC}} - 2\kappa_R^{\text{NC}}$  as a function of the mass of the top quark. (Note that  $\kappa_L^{\text{NC}} = 2\kappa_L^{\text{CC}}$ .)

origin  $(\kappa_L^{\text{NC}}, \kappa_R^{\text{NC}}) = (0, 0)$ , although positive  $\kappa$ 's remain preferred. The reason for this behavior is simply due to the fact that as  $m_t$  increases, the SM value for  $\rho$  ( $\tau$ ) increases in the positive (negative) direction. To summarize this behaviour, we show, respectively, in Figs. 9 and 10 the allowed ranges for  $\kappa_L^{\text{CC}}$  and  $\kappa_L^{\text{NC}} - 2\kappa_R^{\text{NC}}$  as a function of  $m_t$ . An interesting point to mention is that in the global fit analysis the SM ceases to be a solution for  $m_t \gtrsim 200$  GeV. However, with new physics effects, *e.g.*,  $\kappa_L^{\text{CC}} \neq 0$ ,  $m_t$  can be as large as 300 GeV.

In this analysis we concentrated on the physics at the  $Z$  resonance, *i.e.*, at LEP and SLC. Other lower energy observables may as well be used to constrain the non-standard couplings of the top quark to the gauge bosons. In Ref. [41] a constraint on the right-handed charged current ( $\kappa_R^{\text{CC}}$ ) was set using the CLEO measurement of  $b \rightarrow s\gamma$ . The authors concluded that  $\kappa_R^{\text{CC}}$  is well constrained to within a few percent from its SM value ( $\kappa_R^{\text{CC}} = 0$ ). This provides a complementary information to our result because LEP and SLC data are not sensitive to  $\kappa_R^{\text{CC}}$  as compared to  $\kappa_L^{\text{CC}}$ ,  $\kappa_L^{\text{NC}}$ , and  $\kappa_R^{\text{NC}}$ .

## 4 Conclusions

Because top quark is heavy, close to the symmetry-breaking scale, it will be more sensitive than the other light fermions to new physics from the SSB sector. Concentrating on the effects to the low energy data directly related to the SSB sector, we applied the chiral Lagrangian formalism in Lecture One to examine whether the non-standard couplings ( $\kappa$ 's) of the top quark to the gauge bosons ( $W^\pm$  and  $Z$ ). were already strongly constrained by the old (1993) data from LEP and SLC. Surprisingly, we found that to the order of  $m_t^2 \ln \Lambda^2$  only the left-handed neutral current ( $\kappa_L^{\text{NC}}$ ) is somewhat constrained by the precision low energy data. The precision data did impose some correlations among  $\kappa_L^{\text{NC}}$ ,  $\kappa_R^{\text{NC}}$ , and  $\kappa_L^{\text{CC}}$ . Since  $\kappa_R^{\text{CC}}$  does not contribute to the LEP or SLC observables in the limit of taking  $m_b = 0$ , therefore  $\kappa_R^{\text{CC}}$  cannot be constrained by these data. It is however strongly constrained by the complementary process  $b \rightarrow s\gamma$  [41].

In Lecture One we obtained our results by considering a set of Feynman diagrams, derived from the non-linear chiral Lagrangian, whose external lines were the gauge boson lines. The leading corrections (in power of  $m_t$ ) to the low energy observables were found not to vanish in the limit of turning off the weak coupling  $g$  because they originated from being strongly coupled to the SSB sector, *e.g.*, through large Yukawa coupling  $g_t$ . Therefore, our previous results should be able to be reproduced by considering an effective Lagrangian in which all the gauge boson fields are treated as classical fields, namely, they do not contribute to loop calculations. All the loop corrections which do not vanish after taking  $g \rightarrow 0$  can be obtained from calculating

a set of Feynman diagrams only involving the unphysical Goldstone bosons (and probably the Higgs boson) and fermions. In sec. 2 of this lecture we discussed how to relate these two sets of Green's functions for the low energy observables of interest. We showed that by considering a completely different set of Green's functions from that been discussed in Lecture One we obtained exactly the same results.

In sec. 3 we used the new (1994) LEP and SLC data to constrain the non-standard interactions of the top quark to the EW gauge bosons. As compared with the old data from LEP and SLC, the new data tighten the allowed region of the non-standard parameters  $\kappa$ 's by no more than a factor of two. This difference is mainly due to the more precise measurement of  $\Gamma_b$  which turns out to be about  $2\sigma$  higher than the SM prediction and favors a lighter top quark. If the large discrepancy between LEP and SLC data persists, then our model of having non-standard top quark couplings to the gauge bosons is one of the candidates that can accommodate such a difference. Positive values for  $\kappa$ 's are preferred in an effective model, as discussed in Lecture One, where an approximate custodial symmetry is assumed.



# LECTURE THREE:

## Physics of Top Quark At Hadron Colliders

### 1 Introduction

The most important consequence of a heavy top quark is that to a good approximation it decays as a free quark because its lifetime is short and it does not have time to bind with light quarks before it decays [42]. Furthermore, because the heavy top quark has the weak two-body decay  $t \rightarrow bW^+$ , it will analyze its own polarization. Thus we can use the polarization properties of the top quark as additional observables to test the SM and to probe new physics. In the SM, the heavy top quark produced from the usual QCD process, at the Born level, are unpolarized. However, top quarks will have longitudinal polarization if weak effects are present in their production [43]. For instance, the top quark produced from the  $W$ -gluon fusion process is left-handed polarized. With a large number of top quark events, it will be possible to test the polarization effects of the top quarks.

Figure 11: Diagrams contributing to the QCD production of  $q\bar{q}, gg \rightarrow t\bar{t}$

How to detect a SM top quark pair produced via the QCD processes  $q\bar{q}, gg \rightarrow t\bar{t}$ , as shown in Fig. 11, has been extensively studied in the literature [44]. In this lecture we would concentrate on how to detect and study the top quark produced from the single-top quark processes  $q'g(W^+g) \rightarrow qt\bar{b}$ ,  $q'b \rightarrow qt$ ,  $gb \rightarrow W^-t$ , and  $q'\bar{q} \rightarrow W^* \rightarrow t\bar{b}$ . For the single-top productions we will only consider the decay mode of  $t \rightarrow bW^+ \rightarrow b\ell^+\nu$ , with  $\ell^+ = e^+$  or  $\nu^+$ . (The branching ratio for this decay mode is  $\text{Br} = \frac{2}{9}$ .)

The rest of this lecture is organized as follows. In section 2 we discuss the production rates of top quarks at hadron colliders. Following that, we will discuss in sections 3 and 4, respectively, how to measure the mass and the width of the top quark. In section 5 we discuss what we have learned about the couplings of the top quark to the weak gauge bosons and show what can be improved from measuring the production rate of single-top quark event. We will also discuss in section 6 the potential of the Tevatron as a  $\bar{p}p$  collider to probe CP properties of the top quark by simply measuring the single-top quark production rate. Section 7 contains our conclusions. Throughout this lecture we will use  $m_t = 140 \text{ GeV}$  or  $180 \text{ GeV}$  as an example of a light or a heavy top quark for our studies.

Figure 12: Diagrams for various single-top quark processes.

Figure 13: Rate in [pb] for  $q\bar{q}, gg \rightarrow t\bar{t}$ ,  $q'g(W^+g) \rightarrow qt\bar{b}$ ,  $q'\bar{q} \rightarrow W^* \rightarrow t\bar{b}$  and  $gb \rightarrow W^-t$  at various energies of  $\bar{p}p$  colliders.

## 2 The Single-Top Production Mechanism

In this section we consider the production rate of a single-top quark at the Tevatron, the Di-TeV (the upgraded Tevatron) and the LHC (Large Hadron Collider) colliders. In referring to single-top production, unless stated otherwise, we will concentrate only on the positive charge mode (*i.e.*, only including single- $t$ , but not single- $\bar{t}$ ). The colliders we consider are the Tevatron (a  $\bar{p}p$  collider) with the Main Injector at  $\sqrt{S} = 2$  TeV, the Di-TeV (a  $\bar{p}p$  collider) at 4 TeV and the LHC (a  $pp$  collider) at  $\sqrt{S} = 14$  TeV with an integrated luminosity of  $1\text{ fb}^{-1}$ ,  $10\text{ fb}^{-1}$ , and  $100\text{ fb}^{-1}$ , respectively.<sup>11</sup>

A single-top quark signal can be produced from either the  $W$ -gluon fusion process  $q'g(W^+g) \rightarrow qt\bar{b}$  (or  $q'b \rightarrow qt$ ) [45, 46], the Drell-Yan type process  $q'\bar{q} \rightarrow W^* \rightarrow t\bar{b}$  (also known as “ $W^*$ ” production) [47], or the  $Wt$  production via  $gb \rightarrow W^-t$  [48]. The corresponding Feynman diagrams for these processes are shown in Fig. 12

In Figures 13 and 14 we show the total cross sections of these processes for the Tevatron, the Di-TeV and the LHC energies referred to above. For reference we include plots of the cross sections of top quarks as a function of  $m_t$  in both the  $\bar{p}p$  collisions, shown in Figure 13, and  $pp$  collisions, shown in Figure 14. The parton distribution function (PDF) used in our calculation is the leading order set CTEQ2L [49]. We note that taking the  $\Lambda_{\text{QCD}}$  value given in CTEQ2L PDF we obtain  $\alpha_s(M_Z) = 0.127$  which is about 15% larger than the value of 0.110 in CTEQ2M PDF [49]. We found that if we rescale the  $t\bar{t}$  production rates obtained from CTEQ2L PDF with born level amplitudes by the ratio of  $\alpha_s^2(Q, \Lambda_{\text{QCD}})$  from CTEQ2M and that from CTEQ2L, which yields 0.7 for  $Q = M_Z$ , then our total rates are in good agreement with those obtained using NLO PDF and NLO amplitudes [50], see, for example, Ref. [51]. Hereafter we shall use the scaled results for our rates. The constituent cross sections are all calculated at tree level for simplicity to study the kinematics of the top quark and its decay products.

To include the production rates for both single- $t$  and single- $\bar{t}$  events at  $\bar{p}p$  colliders, a factor of 2 should be multiplied to the single- $t$  rates shown in Figures 13 and 14

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<sup>11</sup> In reality, the integrated luminosity can be higher than the ones used here. For instance, with a couple of years of running a 2 TeV Tevatron can accumulate, say,  $10\text{ fb}^{-1}$  luminosity. Similarly, it is not out of question to have a 4 TeV Di-TeV to deliver an integrated luminosity of about  $100\text{ fb}^{-1}$ .

Figure 14: Rate in [pb] for  $q\bar{q}, gg \rightarrow t\bar{t}$ ,  $q'g(W^+g) \rightarrow qt\bar{b}$ ,  $q'\bar{q} \rightarrow W^* \rightarrow t\bar{b}$  and  $gb \rightarrow W^-t$  at various energies of  $pp$  colliders.

Table 1: Rates of the above processes for  $m_t = 140(180)$  GeV. (Branching ratios are not included here.) For  $\sqrt{S} = 2$  TeV and 4 TeV we include rates for a  $\bar{p}p$  machine. At  $\sqrt{S} = 14$  TeV the rates are for a  $pp$  machine. For the single-top rates we only include single- $t$  production.

|                        | Cross Section (pb)                  |  |  |                       |
|------------------------|-------------------------------------|--|--|-----------------------|
| $\sqrt{S}(\text{TeV})$ | $q\bar{q}, gg \rightarrow t\bar{t}$ | $q'g \rightarrow qt\bar{b}$ (or $q'b \rightarrow qt$ ) | $q'\bar{q} \rightarrow W^* \rightarrow t\bar{b}$ | $gb \rightarrow W^-t$ |
| 2                      | 16(4.5)                             | 2(1)   | 0.8(0.3)   | 0.3(0.1)              |
| 4                      | 88(26)                              | 11(7)  | 2.1(0.8)   | 2.9(1.3)              |
| 14                     | 1300(430)                           | 140(100)   | 11(4.6)  | 8.8(3.6)              |

because the parton luminosity for single- $\bar{t}$  production is the same as that for single- $t$ . Similarly, at  $pp$  colliders the rates should be multiplied by  $\sim 1.5$  for the center-of-mass energy ( $\sqrt{S}$ ) of the collider up to  $\sim 4$  TeV, but almost a factor of two at higher energies (say,  $\sqrt{S} \geq 8$  TeV up to about 14 TeV) because the relevant parton luminosities for producing a single- $t$  and a single- $\bar{t}$  event in  $pp$  collisions are different. As shown in Figures 13 and 14 the total rate for single-top production is about the same at  $\bar{p}p$  and  $pp$  colliders for  $\sqrt{S} \geq 8$  TeV because the relevant valence and sea quark parton distributions are about equal for  $100 \text{ GeV} < m_t < 300 \text{ GeV}$ . For smaller  $\sqrt{S}$ , up to  $\sim 4$  TeV, a  $\bar{p}p$  collider is preferred over a  $pp$  collider for heavy top quark production because of its larger parton luminosities. Similarly, for  $t\bar{t}$  pair productions at small  $\sqrt{S}$ , the quark initiated process  $q\bar{q} \rightarrow t\bar{t}$  is more important than the gluon fusion process  $gg \rightarrow t\bar{t}$ . At  $\sqrt{S} \sim 8$  to 14 TeV the  $t\bar{t}$  rate is about the same in  $\bar{p}p$  and  $pp$  collisions because the  $gg \rightarrow t\bar{t}$  subprocess becomes dominant.

For later reference in this lecture, we show the rates of the above processes in Table 1 for  $m_t = 140(180)$  GeV. (Branching ratios are not included here.) For  $\sqrt{S} = 2$  and 4 TeV we include only the rates for a  $\bar{p}p$  machine, whereas at  $\sqrt{S} = 14$  TeV the rates are for a  $pp$  machine. Again, for the single-top rates we only include  $t$  production.

Both in Figures 13 and 14 and Table 1, we have given the cross section of single-top quark produced from either the  $q'g(W^+g) \rightarrow qt\bar{b}$  or  $q'b \rightarrow qt$  processes. From now on, we will refer to this production rate as the rate of the  $W$ -gluon fusion process. The single-top quark produced from the  $W$ -gluon fusion process involves a very important and not yet well-developed technique of handling the kinematics of a *heavy*  $b$  parton inside a hadron. Thus the kinematics of the top quark produced from this process can not be accurately calculated yet. However, the total event rate of the single-top quark production via this process can be estimated using the method proposed in Ref. [52]. The total rate for  $W$ -gluon fusion process involves the  $\mathcal{O}(\alpha^2)$  ( $2 \rightarrow 2$ ) process  $q'b \rightarrow qt$  plus the  $\mathcal{O}(\alpha^2\alpha_s)$  ( $2 \rightarrow 3$ ) process  $q'g(W^+g) \rightarrow qt\bar{b}$  (where the

Figure 15: Feynman diagrams illustrating the subtraction procedure for calculating the total rate for  $W$ -gluon fusion:  $q'b \rightarrow qt \oplus q'g(W^+g) \rightarrow qt\bar{b} \ominus (g \rightarrow b\bar{b} \otimes q'b \rightarrow qt)$ .

Figure 16: Rate in [pb] for single- $t$  production:  $q'b \rightarrow qt$  ( $2 \rightarrow 2$ ),  $q'g \rightarrow qt\bar{b}$  ( $2 \rightarrow 3$ ) and the *splitting* piece  $g \rightarrow b\bar{b} \otimes q'b \rightarrow qt$  in which  $b\bar{b}$  are collinear. The rates are for  $\bar{p}p$  colliders.

gluon splits to  $b\bar{b}$ ) minus the *splitting* piece  $g \rightarrow b\bar{b} \otimes q'b \rightarrow qt$  in which  $b\bar{b}$  are nearly collinear. These processes are shown diagrammatically in Figure 15. The helicity amplitudes and the cross sections for these processes were given in Ref. [53].

The splitting piece is subtracted to avoid double counting the regime in which the  $b$  propagator in the ( $2 \rightarrow 3$ ) process closes to on-shell. This procedure is to resum the large logarithm  $\alpha_s \ln(m_t^2/m_b^2)$  in the  $W$ -gluon fusion process to all orders in  $\alpha_s$  and include part of the higher order  $\mathcal{O}(\alpha^2\alpha_s)$  corrections to its production rate. ( $m_b$  is the mass of the bottom quark.) We note that to obtain the complete  $\mathcal{O}(\alpha^2\alpha_s)$  corrections beyond just the leading log contributions one should also include virtual corrections to the ( $2 \rightarrow 2$ ) process, but we shall ignore these non-leading contributions in this work. Using the prescription described as above we found that the total rate of the  $W$ -gluon fusion process is about a 25% decrease as compared to the ( $2 \rightarrow 2$ ) event rate for  $m_t = 140$  (180) GeV regardless of the energy or the type (*i.e.*,  $pp$  or  $\bar{p}p$ ) of the machine. In Figures 16 and 17 we show the total rate of  $W$ -gluon fusion versus  $m_t$  with scale  $Q = m_t$  as well as a breakdown of the contributing processes at the Tevatron, the Di-TeV and the LHC.

To estimate the uncertainty in the production rate due to the choice of the scale  $Q$  in evaluating the strong coupling constant  $\alpha_s$  and the parton distributions, we show in Figure 18 the scale dependence of the  $W$ -gluon fusion rate. As shown in the figure, although the individual rate from either ( $2 \rightarrow 2$ ), ( $2 \rightarrow 3$ ), or the splitting piece is relatively sensitive to the choice of the scale, the total rate as defined by ( $2 \rightarrow 2$ ) + ( $2 \rightarrow 3$ ) - (splitting piece) only varies by about 30% for  $M_W/2 < Q < 2m_t$  at the Tevatron. At the Di-TeV and the LHC, it varies by about 30% and 10%, respectively. Based upon the results shown in Figure 18, we argue that  $Q < M_W/2$  probably is not a good choice of the relevant scale for the production of the top quark from the  $W$ -gluon fusion process because the total rate rapidly increases by about a factor of 2 in the low  $Q$  regime. In view of the prescription adopted in calculating the total rate, the only relevant scales are the top quark mass  $m_t$  and the virtuality of the  $W$ -line in the scattering amplitudes. Since the typical transverse momentum of the quark ( $q$ ) which comes from the initial quark ( $q'$ ) after emitting the  $W$ -line is about half

Figure 17: Rate in [pb] for single- $t$  production:  $q'b \rightarrow qt$  ( $2 \rightarrow 2$ ),  $q'g \rightarrow qt\bar{b}$  ( $2 \rightarrow 3$ ) and the *splitting* piece  $g \rightarrow b\bar{b} \otimes q'b \rightarrow qt$  in which  $b\bar{b}$  are collinear. The rates are for  $pp$  colliders.

Figure 18: Rate of  $W$ -gluon fusion process versus scale  $Q$  for  $m_t = 180$  GeV and  $\sqrt{S} = 2$  TeV.

of the  $W$ -boson mass, the typical virtuality of the  $W$ -line is about  $M_W/2 \sim 40$  GeV.  $m_b \sim 5$  GeV is thus not an appropriate scale to be used in calculating the  $W$ -gluon fusion rate using our prescription. We note that in the  $(2 \rightarrow 2)$  process the  $b$  quark distribution effectively contains sums to order  $[\alpha_s \ln(Q/m_b)]^n$  from  $n$ -fold collinear gluon emission, whereas the subtraction term (namely, the splitting piece) contains only first order in  $\alpha_s \ln(Q/m_b)$ . Therefore, as  $Q \rightarrow m_b$  the  $(2 \rightarrow 2)$  process picks up only the leading order in  $\alpha_s \ln(Q/m_b)$  and so gets largely cancelled in calculating the total rate. Consequently, as shown in Figure 18, the total rate is about the same as the  $(2 \rightarrow 3)$  rate for  $Q \rightarrow m_b$ . We also note that at  $Q \sim M_W/2$ , the  $(2 \rightarrow 2)$  and  $(2 \rightarrow 3)$  processes have about the same rates, and as  $Q$  increases the  $(2 \rightarrow 2)$  rate, which effectively contains sums of  $[\alpha_s \ln(Q/m_b)]^n$ , gradually increases while the  $(2 \rightarrow 3)$  rate decreases such that the total rate is not sensitive to the scale  $Q$ . It is easy to see also that the total rates calculated via this prescription will not be sensitive to the choice of PDF although each individual piece can have different results from different PDF's, based upon the factorization of the QCD theory [52].

Another single-top quark production mechanism is the Drell-Yan type process  $q'\bar{q} \rightarrow W^* \rightarrow t\bar{b}$ . As shown in Figures 13 and 14, for top quarks with mass on the order of 180 GeV the rate for  $W^*$  production is about one fourth that of  $W$ -gluon fusion at  $\sqrt{S} = 2$  TeV. The  $W^*$  process becomes much less important for heavier top quark. This is because at higher invariant masses  $\hat{s}$  (for producing heavier top quark) of the  $t\bar{b}$  system,  $W^*$  production suffers the usual  $1/\hat{s}$  suppression in the constituent cross section. However, in the  $W$ -gluon fusion process the constituent cross section does not fall off as  $1/\hat{s}$  but flattens out asymptotically to  $1/M_W^2$ . For colliders with higher energies, therefore with large range of  $\hat{s}$ , the  $W^*$  production mechanism for heavy top quarks becomes much less important. However, the kinematics of the top quarks produced from this process are different from those in the  $W$ -gluon fusion events. Moreover, possible new physics may introduce a high mass state (say, particle  $V$ ) to couple strongly with the  $t\bar{b}$  system such that the production rate from  $q'\bar{q} \rightarrow W^* \rightarrow V \rightarrow t\bar{b}$  can largely deviate from the SM  $W^*$  rate.<sup>12</sup> We will however not discuss it in details here because its rate is highly model dependent.

The  $W$ -gluon fusion process becomes more important for heavier top quark. Why? Effectively, the  $W$ -gluon fusion process can be viewed as the scattering of a longitudinal  $W$ -boson ( $W_L$ ) with gluon to produce a top quark and a bottom anti-quark ( $W_L^+ g \rightarrow t\bar{b}$ ) after applying the effective- $W$  approximation [55]. For large  $\hat{s}$  this scattering process is equivalent to  $(\phi^+ g \rightarrow t\bar{b})$  where  $\phi^+$  is the corresponding Goldstone boson of the gauge boson  $W^+$  due to the Goldstone Equivalence Theorem [56, 57]. Since the coupling of  $t\bar{b}\phi$  is proportional to the mass of the top quark, the constituent

<sup>12</sup> This is similar to the speculations made in Ref. [54] for having some high mass resonants in the  $t\bar{t}$  productions.

Figure 19: The lepton+jet decay mode of  $t\bar{t}$  production.

Figure 20: The di-lepton decay mode of  $t\bar{t}$  production.

cross section of the  $W$ -gluon fusion process grows like  $m_t^2/M_W^2$  when  $m_t$  increases. This explains why the  $W$ -gluon fusion rate only decreases slightly as the mass of the top quark increases even though both the parton luminosity and the available phase space decrease for a heavier top quark. In contrast, the  $t\bar{t}$  pair production rate from the QCD processes decreases more rapidly as  $m_t$  increases because the constituent cross section of  $q\bar{q}, gg \rightarrow t\bar{t}$  goes as  $1/\hat{s}$  and the phase space for producing a  $t\bar{t}$  pair is smaller than that for producing a single- $t$ . Therefore, the  $W$ -gluon fusion process becomes more important for the production of a heavy top quark.

Before closing this section, we note that the Effective- $W$  approximation has been the essential tool used in studying the strongly interacting longitudinal  $W$  system to probe the symmetry breaking sector at the supercolliders such as the LHC [20]. By studying the single-top production from the  $W$ -gluon fusion process at the Tevatron, one can learn about the validity of the Effective- $W$  approximation prior to the supercolliders.

### 3 Measuring the Top Quark Mass

By the year 2000, we expect results from the Tevatron (with  $10\text{ fb}^{-1}$ ) and results from LEP-200, giving error of  $\sim 50\text{ MeV}$  on  $M_W$ . Due to Veltman's screening theorem, the low energy data are not sensitive to the mass of the Higgs boson [58]. For a heavy Higgs boson, they can at most depend on  $m_H$  logarithmically up to the one loop level. Therefore, within the SM one needs to also know the mass of the top quark to within  $\sim 5\text{ GeV}$  to start getting useful information on  $m_H$ , with an uncertainty less than a few hundred GeV, through studying radiative corrections to the low energy data which include LEP, SLC, and neutrino experiments [1, 2, 3, 4]. (Of course,  $m_H$  will be measured to some better precision if it is detected in direct production at colliders.)

How accurate can the mass of the top quark be measured at hadron colliders? At hadron colliders,  $m_t$  can be measured in the  $t\bar{t}$  events by several methods [44]. The first method is to use the lepton+jet decay mode of the  $t\bar{t}$  pair, as shown in Fig. 19, by reconstructing the invariant mass of the three jets in the opposite hemisphere from the isolated lepton  $\ell$  ( $= e$  or  $\mu$ ) in  $t \rightarrow bW(\rightarrow \ell\nu)$ , and requiring that two of the three jets reconstruct to a  $W$  and the third be tagged as a  $b$ -jet. The second method is to use the di-lepton decay mode of the  $t\bar{t}$  pair, as shown in Fig. 20, by requiring both  $W$ 's to decay leptonically and for one of the  $b$ 's to decay semileptonically to measure the mass distribution of the non-isolated lepton  $\ell_b$  (from  $b$  decay) and one of the two isolated leptons ( $\ell_1$  and  $\ell_2$  from  $W^\pm$  decay) which is closer to  $\ell_b$ . The third method is to measure the cross section of the di-lepton decay mode of the  $t\bar{t}$  pair. At the LHC, there

Figure 21: Distributions of  $m_{b\ell}$  (solid) and  $m_{\bar{b}\ell}$  (dash) in  $t\bar{t}$  events for a 180 GeV top quark.

will be about  $10^8$   $t\bar{t}$  pairs produced in one year of running for  $m_t < 200$  GeV. With such a large number of events, ATLAS and CMS concluded that  $m_t$  can be measured with a precision of  $\leq 5$  GeV using the first method described above, and about a factor of 2 improvement using the second method [59, 60]. A similar conclusion was also drawn by the CDF and the D0 collaborations for the Tevatron with Main Injector after the upgrade of their detectors [61]. This is remarkable given that the  $t\bar{t}$  cross section at the Tevatron is smaller by about two orders of magnitude as compared with that at the LHC, as shown in Figures 13 and 14.

Next, we would like to discuss how to measure the mass of the top quark in the  $W$ -gluon fusion process. Why do we care? After  $m_t$  is measured in the  $t\bar{t}$  events, we would like to test whether this is a SM top quark. Thus we have to verify its mass measured from other processes, such as in the single-top quark events. Suppose that  $m_t$  is measured from the methods described above in  $t\bar{t}$  events, and the coupling of  $t$ - $b$ - $W$  is not of the SM nature, then we would find that the single-top quark production rate of the  $W$ -gluon fusion process is different from the SM prediction because its production rate is directly proportional to the square of this coupling. (We will discuss more on this point in section 5.) Hence, without knowing the nature of the  $t$ - $b$ - $W$  interactions one can not use the production rates of the single-top quark events to measure  $m_t$ .

Alternatively, we propose two methods to measure  $m_t$  in the single-top quark events. We will refer to them as the fourth and the fifth method. The fourth method is a slight variation of the second method. Instead of measuring the invariant mass of the leptons, we propose to directly measure the invariant mass ( $m_{b\ell}$ ) of the  $\ell$  and  $b$  in  $t \rightarrow bW(\rightarrow \ell\nu)$ . We expect that the efficiency of  $b$  tagging using the displaced vertex is higher for detecting a heavier top quark, and the  $b$  jet energy measurement is better for  $b$  having larger transverse momentum from a heavy top quark decay. Thus  $m_{b\ell}$  can be used to measure the mass of a SM top quark. A detailed Monte Carlo study on the detection of a single-top quark event in hadron collisions was performed in Refs. [46, 53], in which various unique features of the kinematics of the single-top quark and methods in suppressing backgrounds were discussed. We shall however not reproduce that study here. In the  $t\bar{t}$  event there are two  $b$ 's, therefore this method may not work as well as in the single-top event which only contains one  $b$ . However it is not entirely impossible to use this method because, as shown in Figure 21, the sum of the invariant mass distributions of  $b\ell$  and  $\bar{b}\ell$  for a 180 GeV top quark still show a bump near the region that the distribution of  $m_{b\ell}$  peaks. (With a larger sample of  $t\bar{t}$  events one might be able to afford using the electric charge of the soft-lepton from  $b$ -decay to separate  $b$  from  $\bar{b}$  on an event-by-event basis at the cost of the small branching ratio of  $b \rightarrow \mu + X$ , of about 10%.) We will explain in more details how to use  $f_{\text{Long}}$  (the fraction of longitudinal  $W$ -boson from top quark decay), derived from the distribution of  $m_{b\ell}$ , to measure  $m_t$  in section 5.

The fifth method is to reconstruct the invariant mass of the top quark in the  $t \rightarrow bW(\rightarrow \ell\nu)$  decay mode by measuring the missing transverse momentum and choosing a two-fold solution of the longitudinal momentum of the neutrino from the mass constraint of the  $W$  boson. In Refs. [46, 53] we concluded that it is possible to measure  $m_t$  using either of these last two methods to a precision of 5 GeV at the Tevatron ( $\sqrt{S} = 2 \text{ TeV}$ ) with  $1 \text{ fb}^{-1}$  integrated luminosity. We also find that after applying all the kinematical cuts to suppress the dominant background  $W + b\bar{b}$ , at most 10% of  $W^*$  events contribute to the single-top production for a 140 (180) GeV top quark. The SM  $W^*$  production rate is already much smaller than the  $W$ -gluon fusion rate for a heavier top quark, therefore the contribution from the  $W^*$  is not important in our study.

## 4 Measuring the Top Quark Width

As shown in Ref. [62] the intrinsic width of the top quark can not be measured at the high energy hadron collider such as the LHC through the usual QCD processes.<sup>13</sup> For instance, the intrinsic width of a 150 GeV Standard Model top quark is about 1 GeV, and the full width at half maximum of the reconstructed top quark invariant mass (from  $t \rightarrow bW(\rightarrow \text{jets})$  decay mode) is  $\sim 10$  GeV after including the detector resolution effects by smearing the final state parton momenta. Here, the ratio of the measured width and the intrinsic width for a 150 GeV top quark is about a factor of 10. For a heavier top quark, this ratio may be slightly improved because the jet energy can be better measured. (The detector resolution  $\Delta E/E$  for a QCD jet with energy  $E$  is proportional to  $1/\sqrt{E}$ .) A similar conclusion was also given from a hadron level analysis presented in the SDC Technical Design Report which concluded that reconstructing the top quark invariant mass gave a width of 9 GeV for a 150 GeV top quark [63]. Is there a way to measure the top quark width  $\Gamma(t \rightarrow bW^+)$ , say, within a factor of 2 or better, at hadron colliders? Yes, it can in principle be measured in single-top events.

The width  $\Gamma(t \rightarrow bW^+)$  can be measured by counting the production rate of top quarks from the  $W$ - $b$  fusion process which is *equivalent* to the  $W$ -gluon fusion process by a proper treatment of the bottom quark and the  $W$  boson as partons inside the hadron. The  $W$ -boson which interacts with the  $b$ -quark to produce the top quark can be treated as an on-shell boson in the leading log approximation [55, 64]. The result is that even under the approximations considered, a factor of 2 uncertainty in the production rate for this process gives a factor of 2 uncertainty in the measurement of  $\Gamma(t \rightarrow bW^+)$ . This is already much better than what can be measured from the invariant mass distribution of the jets from the decay of top quarks in the  $t\bar{t}$  events

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<sup>13</sup> In Ref. [62] we studied the effects of the QCD radiation in top quark decay (at one loop level) to the measurement of  $m_t$  in  $t\bar{t}$  events produced in hadron collisions. We concluded that the peak position of the  $m_t$  distribution remains about the same as the tree level result, but the shape is different. We also found that the  $m_{b\ell}$  distribution is not sensitive to the QCD radiations in top decay.



produced via the usual QCD processes. More precisely, as argued in section 2 that the production rate of single-top at the Tevatron can probably be known within about 30%, thus it implies  $\Gamma(t \rightarrow bW^+)$  can be measured to about the same accuracy.<sup>14</sup> Therefore, this is an extremely important measurement because it directly tests the couplings of  $t$ - $b$ - $W$ .

$W$ -gluon fusion can also tell us about the Cabibbo-Kobayashi-Maskawa (CKM) matrix element  $|V_{tb}|$ . Assuming only three generations of quarks, the constraints from low energy data together with unitarity of the CKM matrix require  $|V_{tb}|$  to be in 0.9988 to 0.9995 at the 90% confidence level [65]. As noted in Ref. [65] the low energy data do not preclude there being more than three generations of quarks (assuming the same interactions as described by the SM). Moreover, the entries deduced from unitarity might be altered when the CKM matrix is expanded to accommodate more generations. When there are more than three generations the allowed ranges (at 90% CL) of the matrix element  $|V_{tb}|$  can be anywhere between 0 and 0.9995 [65]. Since  $|V_{tb}|$  is directly involved in single-top production via  $W$ -gluon fusion, any deviation from SM value in  $|V_{tb}|$  will either enhance or suppress the production rate of single-top events. It can therefore be measured by simply counting the single-top event rates. For instance, if the single-top production rate is measured to within 30%, then  $|V_{tb}|$  is determined to within 15%.

In conclusion, after the top quark is found, the branching ratio of  $t \rightarrow bW^+(\rightarrow \ell^+\nu)$  can be measured from the ratio of  $(2\ell + jets)$  and  $(1\ell + jets)$  rates in  $t\bar{t}$  events. Because the measured single-top quark event rate is equal to the single-top production rate multiplied by the branching ratio of  $t \rightarrow bW^+(\rightarrow \ell^+\nu)$  for the  $(1\ell + jets)$  mode, and the same  $t$ - $b$ - $W$  couplings appearing in the decay of  $t$  in this process appear also in the production of  $t$ . Thus, a model independent measurement of the decay width  $\Gamma(t \rightarrow bW^+)$  can be made by simply counting the production rate of  $t$  in the  $W$ -gluon fusion process. Should the top quark width be found to be different from the SM expectations, we would then have to look for non-standard decay modes of the top quark. We note that it is important to measure at least one partial width (say,  $\Gamma(t \rightarrow bW^+)$ ) precisely in order to discriminate between different models of new physics, if any. In the SM, the partial width  $\Gamma(t \rightarrow bW^+)$  is about the same as the total width of the top quark at the tree level because of the small CKM matrix element  $|V_{ts}|$ , thus measuring the single-top quark production rate measures the lifetime of the top quark.

## 5 Top Quark Couplings to $W$ Gauge Boson

It is equally important to ask what kind of interactions the  $t$ - $b$ - $W$  vertex might

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<sup>14</sup> Strictly speaking, from the production rate of single-top event, one measures the sum of all the possible partial decay widths, such as  $\Gamma(t \rightarrow bW^+) + \Gamma(t \rightarrow sW^+) + \Gamma(t \rightarrow dW^+) + \dots$ , therefore, this measurement is really measuring the width of  $\Gamma(t \rightarrow XW^+)$  where  $X$  can be more than one particle state as long as it originates from the partons inside proton (or anti-proton). In the SM,  $\Gamma(t \rightarrow bW^+)$  is about equal to the total width of the top quark.

involve [66]. For instance, one should examine the form factors of  $t$ - $b$ - $W$  which result from higher order corrections due to SM strong and/or electroweak interactions. It is even more interesting to examine these form factors to test the plausibility of having *nonuniversal* gauge couplings of  $t$ - $b$ - $W$  due to some dynamical symmetry breaking scenario [27, 34].

The QCD [67] and the electroweak [68] corrections to the decay process  $t \rightarrow bW^+$  in the SM have been done in the literature. The most general operators for this coupling are described by the interaction lagrangian

$$\begin{aligned}
L = & \frac{g}{\sqrt{2}} \left[ W_\mu^- \bar{b} \gamma^\mu (f_1^L P_- + f_1^R P_+) t - \frac{1}{M_W} \partial_\nu W_\mu^- \bar{b} \sigma^{\mu\nu} (f_2^L P_- + f_2^R P_+) t \right] \\
& + \frac{g}{\sqrt{2}} \left[ W_\mu^+ \bar{t} \gamma^\mu (f_1^{L*} P_- + f_1^{R*} P_+) b - \frac{1}{M_W} \partial_\nu W_\mu^+ \bar{t} \sigma^{\mu\nu} (f_2^{R*} P_- + f_2^{L*} P_+) b \right],
\end{aligned} \tag{149}$$

where  $P_\pm = \frac{1}{2}(1 \pm \gamma_5)$ ,  $i\sigma^{\mu\nu} = -\frac{1}{2}[\gamma^\mu, \gamma^\nu]$  and the superscript  $*$  denotes the complex conjugate. In general, the form factors  $f_1^{L,R}$  and  $f_2^{L,R}$  can be complex. Note that in Eq. (149), if there is a relative phase between  $f_1^L$  and  $f_2^R$  or between  $f_1^R$  and  $f_2^L$ , then CP is violated. For instance, in the limit of  $m_b = 0$ , a CP-violating observable will have a coefficient proportional to  $\text{Im}(f_1^L f_2^{R*})$  for a left-handed bottom quark, and  $\text{Im}(f_1^R f_2^{L*})$  for a right-handed bottom quark [66]. (We will discuss more on CP violation in section 6.) If the  $W$ -boson can be off-shell then there are additional form factors such as

$$\partial^\mu W_\mu^- \bar{b} (f_3^L P_- + f_3^R P_+) t + \partial^\mu W_\mu^+ \bar{t} (f_3^{R*} P_- + f_3^{L*} P_+) b, \tag{150}$$

which vanish for an on-shell  $W$ -boson or when the off-shell  $W$ -boson couples to massless on-shell fermions. Here, we only consider on-shell  $W$ -bosons for  $m_t > M_W + m_b$ . At tree level in the SM the form factors are  $f_1^L = 1$  and  $f_1^R = f_2^L = f_2^R = 0$ . These form factors will in general affect the experimental observables related to the top quark, such as the fraction of longitudinal  $W$ 's produced in top quark decays.

The fraction ( $f_{\text{Long}}$ ) of longitudinally polarized  $W$ -bosons produced in the rest frame of the decaying top quarks strongly depends on the form factors  $f_1^{L,R}$  and  $f_2^{L,R}$  [66]. Hence,  $f_{\text{Long}}$  is a useful observable for measuring these form factors. The definition of  $f_{\text{Long}}$  is simply the ratio of the number of longitudinally polarized  $W$ -bosons produced with respect to the total number of  $W$ -bosons produced in top quark decays:

$$f_{\text{Long}} = \frac{\Gamma(\lambda_W = 0)}{\Gamma(\lambda_W = 0) + \Gamma(\lambda_W = -) + \Gamma(\lambda_W = +)} \tag{151}$$

where we use  $\Gamma(\lambda_W)$  to refer to the decay rate for a top quark to decay into a  $W$ -boson with polarization  $\lambda_W$ . ( $\lambda_W = -, +, 0$  denotes a left-handed, right-handed, and longitudinal  $W$ -boson.) Clearly, the polarization of the  $W$ -boson depends on the form factors  $f_1$  and  $f_2$ .<sup>15</sup> Therefore, one can measure the polarization of the  $W$ -boson

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<sup>15</sup>  $f_1^R$  and  $f_1^L$  contribute the same amount of longitudinal  $W$ 's in top quark decays [66].

to measure these form factors. The polarization of the  $W$ -boson can be determined by the angular distribution of the lepton, say,  $e^+$  in the rest frame of  $W^+$  in the decay mode  $t \rightarrow bW^+(\rightarrow e^+\nu)$ . However, the reconstruction of the  $W$ -boson rest frame (to measure its polarization) could be a non-trivial matter due to the missing longitudinal momentum ( $P_z$ ) (with a two-fold ambiguity) of the neutrino ( $\nu$ ) from  $W$  decay. Fortunately, as shown in Eq. (152), one can determine the polarization of the  $W$ -boson without reconstructing its rest frame by using the Lorentz-invariant observable  $m_{be}$ , the invariant mass of  $b$  and  $e$  from  $t$  decay.

The polar angle  $\theta_{e^+}^*$  distribution of the  $e^+$  in the rest frame of the  $W^+$  boson whose z-axis is defined to be the moving direction of the  $W^+$  boson in the rest frame of the top quark can be written in terms of  $m_{be}$  through the following derivation:

$$\begin{aligned}\cos \theta_{e^+}^* &= \frac{E_e E_b - p_e \cdot p_b}{|\vec{\mathbf{p}}_e| |\vec{\mathbf{p}}_b|} \\ &\simeq 1 - \frac{p_e \cdot p_b}{E_e E_b} = 1 - \frac{2m_{be}^2}{m_t^2 - M_W^2}.\end{aligned}\quad (152)$$

The energies  $E_e$  and  $E_b$  are evaluated in the rest frame of the  $W^+$  boson from the top quark decay and are given by

$$\begin{aligned}E_e &= \frac{M_W^2 + m_e^2 - m_\nu^2}{2M_W}, & |\vec{\mathbf{p}}_e| &= \sqrt{E_e^2 - m_e^2}, \\ E_b &= \frac{m_t^2 - M_W^2 - m_b^2}{2M_W}, & |\vec{\mathbf{p}}_b| &= \sqrt{E_b^2 - m_b^2}.\end{aligned}\quad (153)$$

$m_e$  ( $m_\nu$ ) denotes the mass of  $e^+$  ( $\nu_e$ ) for the sake of bookkeeping. The first line in Eq. (152) is exact when using Eq. (153), while the second line of Eq. (152) holds in the limit of  $m_b = 0$ . It is now trivial to find  $f_{\text{Long}}$  by first calculating the  $\cos \theta_{e^+}^*$  distribution then fitting it according to the decay amplitudes of the  $W$ -boson from top quark decay [66]. In what follows we will show how to use the distribution of  $m_{be}$  to measure the mass of the top quark and its couplings to the  $W$ -boson.

In the previous lectures we considered the effective couplings

$$W - t_L - b_L : \frac{g}{2\sqrt{2}} \frac{1 + \kappa_L^{CC}}{2} \gamma_\mu (1 - \gamma_5) \quad (154)$$

and

$$W - t_R - b_R : \frac{g}{2\sqrt{2}} \frac{\kappa_R^{CC}}{2} \gamma_\mu (1 + \gamma_5) \quad (155)$$

derived from an electroweak chiral lagrangian with the symmetry  $SU(2)_L \times U(1)_Y$  broken down to  $U(1)_{em}$ . (Here,  $\kappa_L^{CC} = f_1^L - 1$ , and  $\kappa_R^{CC} = f_1^R$ .) At the Tevatron and the LHC, heavy top quarks are predominantly produced from the QCD process  $gg, q\bar{q} \rightarrow t\bar{t}$  and the  $W$ -gluon fusion process  $qg(Wg) \rightarrow t\bar{b}, \bar{t}b$ . In the former process, one can probe  $\kappa_L^{CC}$  and  $\kappa_R^{CC}$  from the decay of the top quark to a bottom quark and a  $W$  boson. In the latter process, these non-standard couplings can also be measured by simply counting the production rates of signal events with a single  $t$  or  $\bar{t}$ . Let us discuss them in more details as follows.

Figure 22: For a left-handed  $t$ - $b$ - $W$  vertex.

Figure 23: For a right-handed  $t$ - $b$ - $W$  vertex.

### 5.1 From the Decay of Top Quarks

To probe  $\kappa_L^{CC}$  and  $\kappa_R^{CC}$  from the decay of the top quark to a bottom quark and a  $W$  boson, one needs to measure the polarization of the  $W$  boson which can be measured from the distribution of the invariant mass  $m_{b\ell}$ . For a massless  $b$ , the  $W$  boson from top quark decay can only be either longitudinally or left-handed polarized for a left-handed charged current ( $\kappa_R^{CC} = 0$ ). For a right-handed charged current ( $\kappa_L^{CC} = -1$ ) the  $W$  boson can only be either longitudinally or right-handed polarized. (Note that the handedness of the  $W$  boson is reversed for a massless  $\bar{b}$  from  $\bar{t}$  decays.) This is the consequence of helicity conservation, as diagrammatically shown in Figures 22 and 23 for a polarized top quark. In these figures we show the preferred moving direction of the lepton from a polarized  $W$ -boson in the rest frame of a polarized top quark for either a left-handed and a right-handed  $t$ - $b$ - $W$  vertex. As indicated in these figures, the invariant mass  $m_{b\ell}$  depends on the polarization of the  $W$ -boson from the decay of a polarized top quark. Also,  $m_{b\ell}$  is preferentially larger for a pure right-handed  $t$ - $b$ - $W$  vertex than a pure left-handed one. This is clearly shown in Figure 24, in which the peak of the  $m_{b\ell}$  distribution is shifted to the right and the distribution falls off sharply at the upper mass limit for a pure right-handed  $t$ - $b$ - $W$  vertex. In terms of  $\cos\theta_\ell^*$ , their difference is shown in Figure 25. However, in both cases the fraction ( $f_{\text{Long}}$ ) of longitudinal  $W$  from top quark decay is enhanced by  $m_t^2/2M_W^2$  as compared to the fraction of transversely polarized  $W$  [66], namely,

$$f_{\text{Long}} = \frac{\frac{m_t^2}{2M_W^2}}{1 + \frac{m_t^2}{2M_W^2}}. \quad (156)$$

Therefore, for a heavier top quark, it is more difficult to untangle the  $\kappa_L^{CC}$  and  $\kappa_R^{CC}$  contributions. On the other hand, because of the very same reason, the mass of a heavy top quark can be accurately measured from  $f_{\text{Long}}$  irrespective of the nature of the  $t$ - $b$ - $W$  couplings (either left-handed or right-handed).

The QCD production rate of  $t\bar{t}$  is obviously independent of  $\kappa_L^{CC}$  and  $\kappa_R^{CC}$ . (Here we assume the electroweak production rate of  $q\bar{q} \rightarrow A, Z \rightarrow t\bar{t}$  remains small as in the SM.) Let us estimate how well the couplings  $\kappa_L^{CC}$  and  $\kappa_R^{CC}$  can be measured at the Tevatron, the Di-TeV, and the LHC. First, we need to know the production rates of the top quark pairs from the QCD processes. As shown in Table 1, the QCD

Figure 24:  $m_{b\ell}$  distribution for SM top quark (solid) and for pure right-hand  $t$ - $b$ - $W$  coupling of  $tbW$  (dash).

Figure 25:  $\cos \theta_\ell^*$  distribution for SM top quark (solid) and for pure right-hand  $t$ - $b$ - $W$  coupling of  $tbW$  (dash).

Table 2: Results on the accuracy of measuring  $f_1^{L,R}$  for various luminosities. (Only statistical errors are included at the 95% confidence level.)

| Integrated<br>Luminosity<br>$\text{fb}^{-1}$ | Number of<br>reconstructed<br>$t\bar{t}$ events | $\frac{\Delta f_1^L}{f_1^L}$ | $\Delta f_1^R$ | $\frac{\Delta m_t}{m_t}$ |
|--|---|------------------------------|----------------|--------------------------|
| 1  | 200   | 8%                           | $\pm 0.5$      | 4%                       |
| 3  | 600   | 4%                           | $\pm 0.3$      | 2%                       |
| 10   | 2000  | 2%                           | $\pm 0.2$      | 1%                       |

production rate of  $gg, q\bar{q} \rightarrow t\bar{t}$  for a 180 GeV top quark is about 4.5 pb, 26 pb and 430 pb at the Tevatron, the Di-TeV, and the LHC, respectively. For simplicity, let's consider the  $\ell^\pm + \geq 3$  jet decay mode whose branching ratio is  $\text{Br} = \frac{2^2 6}{9^2} = \frac{8}{27}$ , where the charged lepton  $\ell^\pm$  can be either  $e^\pm$  or  $\mu^\pm$ . We assume the experimental detection efficiency ( $\epsilon$ ), which includes both the kinematic acceptance and the efficiency of  $b$ -tagging, to be 15% for the signal event [9]. Let's further assume that there is no ambiguity in picking up the right  $b$  ( $\bar{b}$ ) to combine with the charged lepton  $\ell^+$  ( $\ell^-$ ) to reconstruct  $t$  (or  $\bar{t}$ ), then in total there are  $4.5 \text{ pb} \times 10^3 \text{ pb}^{-1} \times \frac{8}{27} \times 0.15 = 200$  reconstructed  $t\bar{t}$  events to be used in measuring  $\kappa_L^{CC}$  and  $\kappa_R^{CC}$  at  $\sqrt{S} = 2 \text{ TeV}$ . The same calculation at the Di-TeV and the LHC yields 1100 and 19000 reconstructed  $t\bar{t}$  events, respectively. Given the number of reconstructed top quark events, one can fit the  $m_{b\ell}$  distribution to measure  $\kappa_L^{CC}$  and  $\kappa_R^{CC}$ . For example we have done a study for the Tevatron. Let us assume the effects of new physics only modify the SM results ( $f_1^L = 1$  and  $f_1^R = 0$  at Born level) slightly and the form factors  $f_2^{L,R}$  are as small as expected from the usual dimensional analysis [23, 69].<sup>16</sup> We summarize our results on the accuracy of measuring  $f_1^{L,R}$  for various luminosities in Table 2 [70]. (Only statistical errors are included at the 95% confidence level.)

In the same table (*i.e.* Table 2) we also show our estimate on how well the mass of the top quark  $m_t$  can be measured from  $f_{\text{Long}}$ . By definition of  $f_{\text{Long}}$ , for a SM top quark (*i.e.*,  $f_1^L = 1$  and  $f_1^R = 0$ ), the distribution of  $\cos \theta_\ell^*$  has the functional form, in shape, as

$$F(\cos \theta_\ell^*) \sim \left( \frac{1 - \cos \theta_\ell^*}{2} \right)^2 + f_{\text{Long}} \left( \frac{\sin \theta_\ell^*}{\sqrt{2}} \right)^2. \quad (157)$$

<sup>16</sup> The coefficients of the form factors  $f_2^{L,R}$ , assumed to be induced through loop effects, will be a factor of  $\frac{1}{16\pi^2}$  smaller than that of the form factors  $f_1^{L,R}$ .

Therefore,  $f_{\text{Long}}$  can be calculated by fitting with the distribution of  $\cos\theta_\ell^*$ , or equivalently with the distribution of  $m_{b\ell}$ . We prefer to measure  $\kappa_L^{CC}$  and  $\kappa_R^{CC}$  using the distributions of  $m_{b\ell}$  than of  $\cos\theta_\ell^*$  because the former can be directly calculated from the measured momenta of  $b$  and  $\ell$ . However, to convert from the distributions of  $m_{b\ell}$  to  $\cos\theta_\ell^*$ , as given in Eq. (152), the effects from the widths of  $W$ -boson and top quark might slightly distort the distribution of  $\cos\theta_\ell^*$ . (Notice that in the full calculation of the scattering amplitudes the widths of the  $W$ -boson and the top quark have to be included in the Breit-Wigner form to generate a finite event rate.)

However, in reality, the momenta of the bottom quark and the charged lepton will be smeared by detector effects and another problem in this analysis is the identification of the right  $b$  to reconstruct  $t$ . There are three possible strategies to improve the efficiency of identifying the right  $b$ . One is to demand a large invariant mass of the  $t\bar{t}$  system so that  $t$  is boosted and its decay products are collimated. Namely, the right  $b$  will be moving closer to the lepton from  $t$  decay. This can be easily enforced by demanding leptons with a larger transverse momentum. Another is to identify the soft (non-isolated) lepton from  $\bar{b}$  decay (with a branching ratio  $\text{Br}(\bar{b} \rightarrow \mu^+ X) \sim 10\%$ ). The other is to statistically determine the electric charge of the  $b$ -jet (or  $\bar{b}$ -jet) to be  $1/3$  (or  $-1/3$ ) [71]. All of these methods may further reduce the reconstructed signal rate by an order of magnitude. How will these affect our conclusion on the determination of the non-universal couplings  $\kappa_L^{CC}$  and  $\kappa_R^{CC}$ ? It can only be answered by detailed Monte Carlo studies which are yet to be done.

## 5.2 From the Production of Top Quarks

Here we propose another method to measure the couplings  $\kappa_L^{CC}$  and  $\kappa_R^{CC}$  from the production rate of the single-top quark process.

For  $m_t = 180$  GeV, the sum of the production rates of single- $t$  and single- $\bar{t}$  events is about 2 pb and 14 pb for  $\sqrt{S} = 2$  TeV and  $\sqrt{S} = 4$  TeV respectively. The branching ratio of interest is  $\text{Br} = \frac{2}{9}$ . The kinematic acceptance of this event at  $\sqrt{S} = 2$  TeV is about 0.55 [53]. Assuming the efficiency of  $b$ -tagging is about 30%, then there will be  $2 \text{ pb} \times 10^3 \text{ pb}^{-1} \times \frac{2}{9} \times 0.55 \times 0.3 = 75$  events reconstructed for a  $1 \text{ fb}^{-1}$  integrated luminosity. At  $\sqrt{S} = 4$  TeV, the kinematic acceptance of this event is about 0.40 [53] which, from the above calculation, yields about 3700 reconstructed events for  $10 \text{ fb}^{-1}$  integrated luminosity. Based on statistical error alone, this corresponds to a 12% and 2% measurement on the single-top cross section. A factor of 10 increase in the luminosity of the collider can improve the measurement by a factor of 3 statistically. Taking into account the theoretical uncertainties, as discussed in section 2, we examine two scenarios: 20% and 50% error on the measurement of the cross section for single-top production. The results, which are not sensitive to the energies of the colliders considered here (either 2 TeV or 4 TeV), are shown in Figure 26 for a 180 GeV top quark at the Tevatron. We found that  $\kappa_L^{CC}$  and  $\kappa_R^{CC}$  are well constrained inside the region bounded by two (approximate) ellipses. To further determine the sizes of  $\kappa_L^{CC}$  and  $\kappa_R^{CC}$  one needs to study the kinematics of the decay products, such as the charged

Figure 26: Constraint on  $|\kappa_L^{CC}|$  and  $\kappa_R^{CC}$  given 20% and 50% error in measurement of Standard Model rate for  $W$ -gluon fusion. Curves are identical for  $m_t = 140$  GeV and  $m_t = 180$  GeV.

lepton  $\ell$ , of the top quark. Since the top quark produced from the  $W$ -gluon fusion process is almost one hundred percent left-handed (right-handed) polarized for a left-handed (right-handed)  $t$ - $b$ - $W$  vertex, the charged lepton  $\ell^+$  from  $t$  decay has a harder momentum for a right-handed  $t$ - $b$ - $W$  coupling than for a left-handed coupling. (Note that the couplings of light-fermions to  $W$  boson have been well tested from the low energy data to be left-handed as described in the SM.) As shown in Figures 22 and 23, this difference becomes smaller when the top quark is much heavier because the  $W$  boson from the top quark decay tends to be more longitudinally polarized.

A right-handed charged current is absent in a linearly  $SU(2)_L$  invariant gauge theory with massless bottom quark. In this case,  $\kappa_R^{CC} = 0$ , then  $\kappa_L^{CC}$  can be constrained to within about  $-0.08 < \kappa_L^{CC} < 0.03$  ( $-0.20 < \kappa_L^{CC} < 0.08$ ) with a 20% (50%) measurement on the production rate of single-top quark at the Tevatron [34]. (Here we assume the experimental data agrees with the SM prediction within 20% (50%).) This means that if we interpret  $(1 + \kappa_L^{CC})$  as the CKM matrix element  $|V_{tb}|$ , then  $|V_{tb}|$  can be bounded as  $|V_{tb}| > 0.9$  (or 0.75) for a 20% (or 50%) measurement on the single-top production rate.

Before closing this section, we would remark that in the previous lectures and in the Refs. [34] and [41] some bounds on the couplings of  $\kappa_L^{CC}$  and  $\kappa_R^{CC}$  were obtained by studying the low energy data with the assumption that the effects of new physics at low energy can only modify the couplings of  $\kappa_L^{CC}$  and  $\kappa_R^{CC}$  but not introduce any other light fields in the effective theory. However, nature might not behave exactly in this way. It is possible that some light fields may exist just below the TeV scale, then the bounds obtained from Refs. [34] and [41] may no longer hold. Thus, it is important to have the direct measurement on all the form factors listed in Eq. (149) from the production of top quarks, in spite of the present bounds on  $\kappa$ 's derived from radiative corrections to low energy data.

## 6 Probing CP Properties in Top Quarks

It is known that explicit CP violation requires the presence of both the CP non-conserving vertex and the complex structure of the physical amplitude. Due to the origin of this complex structure, the possible CP-violating observables can be separated into two categories. In the first category, this complex structure comes from the absorptive part of amplitude due to the final state interactions. In the second category, this complex structure does not arise from the absorptive phase but from the correlations in the kinematics of the initial and final state particles involved in the physical process. Hence, it must involve a triple product correlation (*i.e.*, a Levi-Civita tensor).

To distinguish the symmetry properties between these two cases, we introduce the transformation  $\hat{T}$ , as defined in Ref. [72], which is simply the application of time reversal to all momenta and spins without interchanging initial and final states. The CP-violating observables in the first category are CP-odd and  $\text{CPT}\hat{T}$ -odd, while those in the second category are CP-odd and  $\text{CPT}\hat{T}$ -even. Of course, both of them are CPT-even.

As an illustration of the above two categories, we consider the CP-violating observables for the decay of the top quark. Consider the partial rate asymmetry

$$\mathcal{A}_{bW} \equiv \frac{\Gamma(t \rightarrow bW^+) - \Gamma(\bar{t} \rightarrow \bar{b}W^-)}{\Gamma(t \rightarrow bW^+) + \Gamma(\bar{t} \rightarrow \bar{b}W^-)}. \quad (158)$$

This observable clearly violates CP and  $\text{CPT}\hat{T}$  and therefore belongs to the first category. We note that because of CPT invariance, the total decay width of the top quark  $\Gamma(t)$  has to equal the total decay width of the top anti-quark  $\Gamma(\bar{t})$ . Thus, any non-zero  $\mathcal{A}_{bW}$  implies that there exists a state (or perhaps more than one state)  $X$  such that  $t$  can decay into  $X$ , and  $\bar{t}$  into  $\bar{X}$ . The absorptive phase of  $t \rightarrow bW^+$  is therefore generated by re-scattering through state  $X$ , *i.e.*,  $t \rightarrow X \rightarrow bW^+$ , where  $X \neq bW^+$  because the final state interaction should be off-diagonal [73].

Next, let's consider the observable of the second category. In the decay of  $t \rightarrow bW^+(\rightarrow \ell^+\nu_\ell)$ , for a polarized  $t$  quark, the time-reversal invariance (T) is violated if the expectation value of

$$\vec{\sigma}_t \times \vec{p}_b \cdot \vec{p}_{\ell^+} \quad (159)$$

is not zero [66]. Assuming CPT invariance, this implies CP is violated. Therefore, this observable is CP-odd but  $\text{CPT}\hat{T}$ -even. A non-vanishing triple product observable, such as that in Eq. (159), from the decay of the top quark violates T, however it may be entirely due to final state interaction effects without involving any CP-violating vertex. To construct a truly CP-violating observable, one must combine information from both the  $t$  and  $\bar{t}$  quarks. For instance, the difference in the expectation values of  $\vec{\sigma}_t \times \vec{p}_b \cdot \vec{p}_{\ell^+}$  and  $\vec{\sigma}_{\bar{t}} \times \vec{p}_{\bar{b}} \cdot \vec{p}_{\ell^-}$  would be a true measure of an intrinsic CP violation.

There have been many studies on how to measure the CP-violating effects in the  $t\bar{t}$  system produced in either electron or hadron collisions. (For a review, see a recent paper in Ref. [74].) At hadron colliders, the number of  $t\bar{t}$  events needed to measure a CP-violating effect of the order of  $10^{-3} - 10^{-2}$  is about  $10^7 - 10^8$ . To examine the potential of various current and future hadron colliders in measuring the CP-violating asymmetries, we estimate the total event rates of  $t\bar{t}$  pairs for a 180 GeV SM top quark produced at these colliders. At the Tevatron, the Di-TeV, and the LHC, an integrated luminosity of 10, 100, and 100  $\text{fb}^{-1}$  will produce about  $4.5 \times 10^4$ ,  $2.6 \times 10^6$ , and  $4.3 \times 10^7$   $t\bar{t}$  pairs, respectively, as given in Table 1. Therefore, the LHC would be able to probe the CP asymmetry of the top quark at the level of a few percent. A similar number of the  $t\bar{t}$  pairs is required in electron collision to probe the CP asymmetry at the same level. Thus, for a  $\sqrt{S} = 500 \text{ GeV}$   $e^-e^+$  collider, an



integrated luminosity of about  $10^4 - 10^5 \text{ fb}^{-1}$  has to be delivered. This luminosity is at least a factor of 100 higher than the planned next linear colliders. We note that although the initial state in a pp collision (such as at the LHC) is not an eigenstate of a CP transformation, these CP-odd observables can still be defined as long as the production mechanism is dominated by  $gg$  fusion. This is indeed the case for  $t\bar{t}$  pair productions at the LHC.

In the SM, the top quark produced via the  $W$ -gluon fusion process is about one hundred percent left-handed (longitudinally) polarized. Given a polarized top quark, one can use the triple product correlation, as defined in Eq. (159), to detect CP violation of the top quark. For a polarized top quark, one can either use  $\vec{\sigma}_t \times \vec{p}_b$  or  $\vec{p}_t^{\text{Lab}} \times \vec{p}_b$  to define the decay plane of  $t \rightarrow bW(\rightarrow \ell^+\nu)$ . Obviously, the latter one is easier to implement experimentally. Define the asymmetry to be

$$\mathcal{A}_{io} \equiv \frac{N(\ell^+ \text{ out of the decay plane}) - N(\ell^+ \text{ into the decay plane})}{N(\ell^+ \text{ out of the decay plane}) + N(\ell^+ \text{ into the decay plane})}. \quad (160)$$

If  $\mathcal{A}_{io}$  is not zero, then the time-reversal  $T$  is not conserved, therefore CP is violated for a CPT invariant theory. Due to the missing momentum of the neutrino from the decay of the  $W$ -boson, it is difficult to reconstruct the azimuthal angle ( $\phi_W$ ) of the  $W$ -boson from the decay of the top quark. Once the angle  $\phi_W$  is integrated over, the transverse polarization of the top quark averages out, and only the longitudinal polarization of the top quark contributes to the asymmetry  $\mathcal{A}_{io}$ . Thus, the asymmetry  $\mathcal{A}_{io}$  can be used to study the effects of CP violation in the top quark, which in the SM is about one hundred percent left-handed (longitudinally) polarized as produced from the  $W$ -gluon fusion process. To apply the CP-violating observable  $\mathcal{A}_{io}$ , one needs to reconstruct the directions of both the  $t$  and  $b$  quarks. It has been shown in Ref. [75] that it takes about  $10^7 - 10^8$  single-top events to detect CP violation at the order of  $\sim 10^{-3} - 10^{-2}$ .

For  $m_t = 180 \text{ GeV}$  at the Tevatron, the Di-TeV, and the LHC, an integrated luminosity of 10, 100, and  $100 \text{ fb}^{-1}$  will produce about  $2 \times 10^4$ ,  $1.4 \times 10^6$ , and  $2 \times 10^7$  single- $t$  or single- $\bar{t}$  events, respectively, Table 1. At the NLC, the single top quark production rate is much smaller. For a 2 TeV electron collider, the cross sections for  $e^-e^+ \rightarrow e^-\bar{\nu}_e t\bar{b}$  and  $e^+\gamma \rightarrow \bar{\nu}_e t\bar{b}$  are 8 fb and 60 fb, respectively [76]. Hence, it will be extremely difficult to detect CP violation effects at the order of  $\leq 10^{-2}$  in the single-top events produced in electron collisions.

A few comments are in order. First, to extract the *genuine* CP-violating effects, we need to study the difference in the asymmetry  $\mathcal{A}_{io}$  measured in the single- $t$  and single- $\bar{t}$  events because the time-reversal violation in  $\mathcal{A}_{io}$  of the  $t$  (or  $\bar{t}$ ) alone could be generated by final state interactions without CP-violating interactions. Second, the detection efficiency for this method is not close to one, so a good understanding of the kinematics of the decay products and how the detector works are needed to make this method useful.

The asymmetry  $\mathcal{A}_{io}$  belongs to the second category of CP-violating observables, and is CP-odd and  $\text{CPT}$ -even. Here, let's consider another asymmetry  $\mathcal{A}_t$  which

belongs to the first category of CP-violating observables, and is CP-odd and  $\text{CP}\hat{\text{T}}$ -odd. Using  $\mathcal{A}_t$  for detecting CP-violating effects is to make use of the fact that  $\bar{p}p$  is a CP eigenstate; therefore, the difference in the production rates for  $\bar{p}p \rightarrow tX$  and  $\bar{p}p \rightarrow \bar{t}X$  is a signal of CP violation. This asymmetry is defined to be

$$\mathcal{A}_t \equiv \frac{\sigma(\bar{p}p \rightarrow tX) - \sigma(\bar{p}p \rightarrow \bar{t}X)}{\sigma(\bar{p}p \rightarrow tX) + \sigma(\bar{p}p \rightarrow \bar{t}X)} . \quad (161)$$

As discussed in section 5, the production rate of  $\bar{p}p \rightarrow tX$  is proportional to the decay rate of  $t \rightarrow bW^+$ , and the rate of  $\bar{p}p \rightarrow \bar{t}X$  is proportional to the rate of  $\bar{t} \rightarrow \bar{b}W^-$ . This implies that  $\mathcal{A}_t = \mathcal{A}_{bW}$ , cf. Eq. (158). There have been quite a few models studied in the literature about the asymmetry in  $\mathcal{A}_{bW}$ . For instance, in the Supersymmetric Standard Model where a CP-violating phase may occur in the left-handed and right-handed top-squark,  $\mathcal{A}_{bW}$  can be as large as a few percent depending on the details of the parameters in the model [77].

Next, let's examine how many top quark events are needed to detect a few percent effect in the CP-violating asymmetry  $\mathcal{A}_t$ . Consider  $t \rightarrow bW^+ \rightarrow b\ell^+\nu$ , where  $\ell = e$  or  $\mu$ . Define the branching ratio  $B_W$  as the product of  $\text{Br}(t \rightarrow bW^+)$  and  $\text{Br}(W^+ \rightarrow \ell^+\nu)$ , where  $\text{Br}(W^+ \rightarrow \ell^+\nu)$  is 2/9. ( $\text{Br}(t \rightarrow bW^+)$  depends on the details of a model, and is almost 1 in the SM.) Let us assume that the efficiency of  $b$ -tagging ( $\epsilon_{\text{btag}}$ ) is about 30%, and the kinematic acceptance ( $\epsilon_k$ ) of reconstructing the single-top event,  $\bar{p}p \rightarrow tX \rightarrow bW^+X \rightarrow b\ell^+\nu X$ , is about 50%.<sup>17</sup> The number of single- $t$  and single- $\bar{t}$  events needed to measure  $\mathcal{A}_t$  is

$$\mathcal{N}_t = \frac{1}{B_W \epsilon_{\text{btag}} \epsilon_k} \left( \frac{1}{\mathcal{A}_t} \right)^2 . \quad (162)$$

Thus, to measure  $\mathcal{A}_t$  of a few percent,  $\mathcal{N}_t$  has to be as large as  $\sim 10^6$ , which corresponds to an integrated luminosity of  $100 \text{ fb}^{-1}$  at the Di-TeV.

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<sup>17</sup> This was obtained from a Monte Carlo study performed in Ref. [53].

## 7 Discussions and Conclusions

We discussed the physics of top quark production and decay at hadron colliders, such as the Tevatron, the Di-TeV and the LHC. We showed how to measure the mass and the width of the top quark, produced from either a single-top or a  $t\bar{t}$  pair process, using the invariant mass distribution of  $m_{b\ell}$ . It has been shown in Ref. [62] that the distribution of  $m_{b\ell}$  is not sensitive to radiative corrections from QCD interactions, thus it can be reliably used to test the polarization of the  $W$ -boson from  $t$  decay, therefore test the polarization of the top quark from the production mechanism. We also discussed how well the couplings of  $t$ - $b$ - $W$  vertex can be studied to probe new physics, and how well the CP properties of the top quark can be tested in electron or hadron colliders.

In Ref. [46] we showed that an almost perfect efficiency for “kinematic  $b$  tagging” can be achieved due to the characteristic features of the transverse momentum and rapidity distributions of the spectator quark which emitted the virtual  $W$ . In addition, the ability of performing  $b$ -tagging using a vertex detector increases the detection efficiency of a heavy top quark produced via the  $W$ -gluon fusion process. A detailed Monte Carlo study was performed in Ref. [53] to show that this process is extremely useful at the Tevatron with Main Injector. For an integrated luminosity of  $1 \text{ fb}^{-1}$ , there will be about 105 (75) single- $t$  or single- $\bar{t}$  events reconstructed in the lepton+jet mode for  $m_t = 140$  (180) GeV at  $\sqrt{S} = 2 \text{ TeV}$ . (The branching ratio of  $W \rightarrow e$ , or  $\mu$  is included, and The  $b$ -quark tagging efficiency is assumed to be 30% for  $P_t^b > 30 \text{ GeV}$  with no misidentifications of a  $b$ -jet from other QCD jets.) The dominant background process is the electroweak-QCD process  $W + b\bar{b}$  whose rate is about 60%(80%) of the signal rate in the end of the analysis. The  $t\bar{t}$  events are not as important to our study. The results for  $\sqrt{S} = 4 \text{ TeV}$  at the Di-TeV and for  $\sqrt{S} = 14 \text{ TeV}$  at the LHC were also given in Ref. [53].

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